A stochastic data envelopment analysis model using a common set of weights and the ideal point concept

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Abstract: The efficiency scores of the decision making units (DMUs) in conventional data envelopment analysis (DEA) are between zero and one and generally several DMUs result in having efficiency scores of one. These models generally only rank the inefficient DMUs and not the efficient ones. In addition, conventional DEA models assume that inputs and outputs are measured precisely on a ratio scale. However, the observed values of the input and output data in real-life problems are often imprecise. In this paper, we propose a common set of weights (CSW) model for ranking the DMUs with the stochastic data and the ideal point concept. The proposed method minimises the distance between the evaluated DMUs and the ideal DMU. We also present a numerical example to demonstrate the applicability of the proposed model and exhibit the efficacy of the procedures.

Keywords: data envelopment analysis; DEA; stochastic data; common set of weights; CSWs; ideal point.

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1 Introduction

Data envelopment analysis (DEA) is a non-parametric approach proposed by Charnes et al. (1978) and later developed by Banker et al. (1984) for measuring the efficiency of a homogeneous group of decision making units (DMUs) based on multiple inputs and outputs from observed data (Jahanshahloo et al., 2008, 2010a, 2010b; Despotis and Smirlis, 2002; Rezaie et al., 2009). DEA provides efficiency scores for each DMU as ratios of a weighted sum of the outputs to a weighted sum of the inputs. The basic DEA groups the DMUs into efficient and inefficient units.

The original DEA models (Charnes et al., 1994) assume that inputs and outputs are measured by exact values based upon well-defined factors (Despotis and Smirlis, 2002). However, the efficiency evaluation process sometimes involves stochastic estimations because of the uncertainties inherent in many real-life problems. The stochastic framework assumes that the DMUs are characterised by distribution moments (Brazdik, 2004). Stochastic input and output variations in DEA (SDEA) have been studied by many researchers including: Land et al. (1993), Olesen and Petersen (1995), Huang and Li (1996), Li (1998), Morita and Seiford (1999), Cooper et al. (1996, 1998, 2002, 2004), Bruni et al. (2009), Khodabakhshi et al. (2010), Khodabakhshi and Asgharian (2009), Asgharian et al. (2010), Khodabakhshi (2009, 2010, 2011), Wu and Lee (2010) and Lotfi et al. (2012).

One problem that has been frequently studied in the DEA literature has been the lack of discrimination in many performance evaluation problems. When DEA models are used to evaluate the relative efficiency of a DMU, the resulting efficiency scores lie between zero and one and in many cases will be equal to one (Rezaie et al., 2009; Jahanshahloo et al., 2008). DEA divides them into two categories; efficient DMUs and inefficient DMUs. Although a ranking for inefficient DMUs is given, efficient DMUs cannot be ranked (Jahanshahloo et al., 2010b).

The lack of discrimination between efficient DMUs is considered an important problem in DEA. In response, several researchers have sought to improve the differential capabilities of DEA and to fully rank both efficient and inefficient DMUs (Jahanshahloo et al., 2008). Some of them are more general such as Anderson and Petersen (1993) and Mehrabian et al. (1999). Anderson and Petersen (1993) developed a modified DEA model based on comparison of efficient DMUs relative to a reference technology. They proposed a framework for ranking efficient units and facilitated comparison with rankings based on parametric methods. Mehrabian et al. (1999) showed that the modified DEA model proposed by Andersen and Petersen (1993) breaks down in some cases and can be unstable when one of the DMUs has a relatively small value for some of its inputs. They proposed an alternative efficiency measure based on a different optimisation problem and eliminated the difficulties with Andersen and Petersen's (1993) method.

Cook et al. (1992) also showed that a relatively large number of the DMUs are credited with an efficiency score of 1 in DEA with no clear means of discriminating among such units. They studied various conditions that are imposed on the multipliers in a DEA problem and suggested an approach for breaking ties on the frontier in each case. Jahanshahloo et al. (2007) introduced a new ranking system for extreme efficient DMUs based on the omission of the efficient DMUs from the reference set of the inefficient DMUs. Liu and Peng (2008) proposed a methodology to determine one common set of weights (CSWs) for the performance indices of the efficient DMUs in DEA. They then ranked these DMUs according to their efficiency scores weighted by the CSWs. Rezaie et al. (2009) used the concept of the ideal DMU to obtain an efficiency interval consisting of evaluations from both the optimistic and the pessimistic viewpoints. Using this concept, they improve the efficiencies of the DMUs so that their lower bounds become large enough to attain the maximum value 1. In order to improve the lower bound of the efficiency interval, they define different ideal points for different DMUs. Jahanshahloo et al. (2010b) proposed two ranking methods. In the first method, they defined an ideal line and determined a CSWs for efficient DMUs to obtain a new efficiency score and rank the DMUs. In the second method, they defined a special line and ranked the DMUs by comparing all efficient ones to this line.

In this paper, we use the ideal point concept to rank the efficiency of the DMUs. We develop a linear programming model using a CSWs and use stochastic data to obtain the efficiency scores of the DMUs. The remainder of this paper is organised as follows. In Section 2, we introduce the CCR model in the SDEA. In Section 3, we propose the CSW-SDEA model using the ideal point concept. In Section 4, we present a numerical example to demonstrate the applicability of the proposed model and exhibit the efficacy of the procedures. Conclusions and future research are given in Section 5.

2 Stochastic DEA

Assume that there are a set of n DMUs, and each DMU_j, (j = 1, ……, n) produces s different outputs using m different inputs which are denoted as x_{ij}, (i = 1, ……, m) and y_{rj}, (r = 1, …, s), respectively. Here x_{ij} and y_{rj} are all positive deterministic elements (Charnes et al., 1978).

Now assume a stochastic framework with $\tilde{x}_j = (\tilde{x}_{1j}, \ldots, \tilde{x}_{mj})^t$ and $\tilde{y}_j = (\tilde{y}_{1j}, \ldots, \tilde{y}_{sj})^t$ denoting the random input and output data for DMU_j. Let us further assume that $x_j = (x_{1j}, \ldots, x_{mj})^t$ and $y_j = (y_{1j}, \ldots, y_{sj})^t$ represent the corresponding vectors of expected values of the inputs and outputs for DMU_j. Suppose that all the input and output components are jointly normally distributed; then, we obtain the following deterministic model which is a CCR stochastic super-efficiency model, where slack variables are all excluded from the objective function using the properties of a normal distribution, and replacing non-negative variables w_i^I and w_r^o, respectively:

$$
\begin{aligned}
& Min\, \theta_o^s \\
& \text{s.t.} \\
& \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j x_{ij} + s_i^- - \varphi^{-1}(\alpha) w_i^I = \theta_o^s x_{io}, \quad i = 1, \ldots, m \\
& y_{ro} - \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j y_{rj} + s_r^+ - \varphi^{-1}(\alpha) w_r^o = 0, \quad r = 1, \ldots, s \\
& \left(w_i^I\right)^2 = \sum_{j\neq 0}\sum_{k\neq 0} \lambda_j \lambda_k\, Cov\left(\tilde{x}_{ij}, \tilde{x}_{ik}\right) + 2\theta_o^s \sum_{j\neq 0} \lambda_j\, Cov\left(\tilde{x}_{ij}, \tilde{x}_{io}\right) + (\theta_o^s)^2\, Var\left(\tilde{x}_{io}\right) \\
& \left(w_r^o\right)^2 = \sum_{k\neq 0}\sum_{j\neq 0} \lambda_k \lambda_j\, Cov\left(\tilde{y}_{rk}, \tilde{y}_{rj}\right) + 2\sum_{k\neq 0} \lambda_k\, Cov\left(\tilde{y}_{rk}, \tilde{y}_{ro}\right) + Var\left(\tilde{y}_{ro}\right) \\
& \lambda_j, s_r^+, s_i^-, w_i^I, w_r^o \geq 0
\end{aligned}
\tag{1}
$$

where φ is the cumulative distribution function (cdf) of a standard normal random variable and φ^{-1} is its inverse. It is assumed that x_{ij} and y_{rj} are the means of the input and output variables, respectively (Hosseinzadeh Lotfi et al., 2007; Hosseinzadeh Lotfi et al., 2010).

Definition 1 [Stochastic efficiency according to model (1)]: DMU_o is stochastically efficient if and only if the following conditions are satisfied:

1 $\theta_o^* = 1$

2 slack variables are zero in all alternative optimal solutions.

Also we propose a multiplier version of model (1) in the following:

$$\max \sum_{r=1}^{s} u_r \bar{\tilde{y}}_{ro}$$

s.t.

$$\sum_{r=1}^{s} u_r \bar{\tilde{y}}_{rj} - \sum_{i=1}^{m} v_i \bar{\tilde{x}}_{ij} - \varphi^{-1}(\alpha) w_j \leq 0, \qquad j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m} v_i \bar{\tilde{x}}_{io} + \varphi^{-1}(\alpha) w_j = 1 \qquad (2)$$

$$w_j^2 = \sum_{r=1}^{s}\sum_{k=1}^{s} u_r u_k Cov(\tilde{y}_{rj}, \tilde{y}_{kj}) + \sum_{i=1}^{m}\sum_{l=1}^{m} v_i v_l Cov(\tilde{x}_{ij}, \tilde{x}_{lj}) - 2\sum_{r=1}^{s}\sum_{i=1}^{m} u_r v_i Cov(\tilde{y}_{rj}, \tilde{x}_{ij})$$

$$u_r \geq \varepsilon \geq 0 \qquad r = 1, 2, \ldots, s$$

$$v_i \geq \varepsilon \geq 0 \qquad i = 1, 2, \ldots, m$$

3 Proposed model

In this section, we propose a new model to extend the existing SDEA model for generating common weights using the concept of an *ideal* DMU. Using this concept, we obtain and compare the efficiency scores using different approaches.

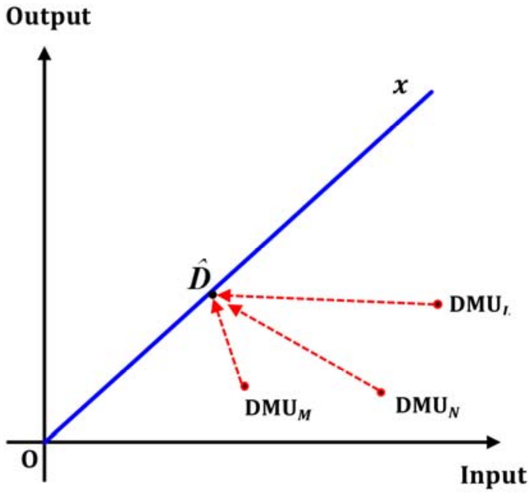
Definition 2: Assume that there are n DMUs, where each DMU$_j$, (j = 1, ……, n) uses m different inputs that have a normal distribution, $\tilde{x}_{ij}, (i = 1, \ldots, m)$ to produce s different outputs that also have a normal distribution $\tilde{y}_{rj}$, $(r = 1, \ldots, s)$. All the data are required to be positive as is assumed in traditional DEA (Charnes et al., 1978).

The virtual ideal DMU is a DMU which minimises the inputs of the DMUs and maximises the outputs of the DMUs. Generally, if we represent the ideal DMU as $\hat{D} = (\hat{X}, \hat{Y})$; then, we have $\hat{x}_i = min\{\bar{\tilde{x}}_{ij} \mid j = 1, \ldots, n\}, (i = 1, \ldots, m)$ and $\hat{y}_r = max\{\bar{\tilde{y}}_{rj} \mid j = 1, \ldots, n\}$, $(r = 1, \ldots, s)$ where $\bar{\tilde{y}}_{rj} = \frac{\sum \tilde{y}_{rj}}{n}$ and $\bar{\tilde{x}}_{ij} = \frac{\sum \tilde{x}_{ij}}{n}$ for each DMU.

In Figure 1, the vertical and horizontal axes represent the weighted sum of s outputs and the weighted sum of m inputs, respectively. Line ‘ox’ is an ideal line which represents the constraint that the weighted sum of s outputs equals to the weighted sum of m inputs, and $\hat{D} = \left(\sum_{i=1}^{m} \hat{x}_i v_i', \sum_{r=1}^{s} \hat{y}_r u_i' \right)$ is an ideal DMU. For any DMU, given a set of weights u_r, (r = 1, …, s) and v_i, (i = 1, …, m); then, the virtual gaps, between points M and $\hat{D}$ on the horizontal axes and vertical axes, are denoted as $\sum_{j=1}^{n} v_M x_{Mj} - \sum_{j=1}^{n} v_M \hat{x}_{min}$ and

$\sum_{j=1}^{n} u_M \hat{y}_{max} - \sum_{r=1}^{s} u_M y_{Mj}$, respectively. The gaps for points N and L will be calculated in a similar manner. We also observe a total virtual gap from the ideal point.

Figure 1 A gap analysis depiction showing DMUs below the virtual ideal DMU (see online version for colours)



We want to determine an optimal set of weights $u_r^*(r = 1, \ldots, s)$ and $v_i^*(i = 1, \ldots, m)$ such that all points below the ideal line should be as close as possible to their ideal point $(\hat{D})$. In other words, by adopting the optimal weights, we can minimise the total virtual gaps to the ideal point.

As for the constraint, the numerator is the weighted sum of the outputs plus the vertical gap and the denominator is the weighted sum of the inputs minus the horizontal virtual gap. Setting the right hand side of the constraints to 1 means that the ideal DMU is reached. Therefore, we have the following multiplier linear programming model:

$$
\begin{aligned}
& Min \sum_{j=1}^{n} \left[\sum_{i=1}^{m} v_i \bar{\tilde{x}}_{ij} - \sum_{i=1}^{m} v_i \hat{x}_{min} \right] + \sum_{j=1}^{n} \left[\sum_{r=1}^{s} u_r \hat{y}_{max} - \sum_{r=1}^{s} u_r \bar{\tilde{y}}_{rj} \right] \\
& \text{s.t.} \\
& \sum_{i=1}^{m} v_i \hat{x}_{min} = 1 \\
& \sum_{r=1}^{s} u_r \hat{y}_{max} = 1 \qquad\qquad (3) \\
& w_j{}^2 = \sum_{r=1}^{s} \sum_{k=1}^{s} u_r u_k Cov\left(\tilde{y}_{rj}, \tilde{y}_{kj}\right) + \sum_{i=1}^{m} \sum_{l=1}^{m} v_i v_l Cov\left(\tilde{x}_{ij}, \tilde{x}_{lj}\right) - 2 \sum_{r=1}^{s} \sum_{i=1}^{m} u_r v_i Cov\left(\tilde{y}_{rj}, \tilde{x}_{ij}\right) \\
& v_i, u_r, v_i' \geq \varepsilon \geq 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s
\end{aligned}
$$

From model (3), it is found that the distance between $\hat{D}$ and DMU$_j$ is defined as

$$\left(\sum_{i=1}^{m} v_i x_{ij} - \sum_{i=1}^{m} v_i \hat{x}_{min}\right) + \left(\sum_{r=1}^{s} u_r \hat{y}_{max} - \sum_{r=1}^{s} u_r y_{rj}\right).$$

The purpose of model (3) is to obtain an optimal solution (v_i^*, u_r^*) to minimise the total distances between all the DMUs and $\hat{D}$ (Mavi et al., 2013).

Next, we find the efficiency of each DMU with optimal weights. If a DMU$_j$ is on the ideal point; then, we use the definition of the CSW efficiency score proposed by Liu and Peng (2008) as follows (also see Jahanshahloo et al., 2010a; Sun et al., 2013):

$$\mu_j^* = \frac{\sum_{r=1}^{s} y_{rj} u_r^*}{\sum_{i=1}^{n} x_{ij} v_i^*} \tag{4}$$

4 Numerical example

In this section, we present a numerical example to demonstrate the applicability of the proposed model and exhibit the efficacy of the procedures. Let us consider five bank branches with two stochastic inputs and two stochastic outputs. The two inputs and the two outputs are defined as follows: *X*1 is 'operating budget', *X*2 is 'number of employees', *Y*1 is 'cash deposits' and *Y*2 is 'earned profits' of bank. Assuming that the inputs and outputs are stochastic with a normal distribution, the means of the inputs and outputs are presented in Table 1.

Table 1 Mean of the inputs and outputs

Inputs and outputs			*DMUs (bank branches)*				
			1	*2*	*3*	*4*	*5*
Inputs	Operating budget	$\bar{\tilde{x}}_{1j}$	6,214.70	4,937.71	16,264.77	3,187.22	109,92.61
	Number of employees	$\bar{\tilde{x}}_{2j}$	13.14	12.42	13.86	16.50	11.88
Outputs	Cash deposits	$\bar{\tilde{y}}_{1j}$	262,125.65	180,786.77	271,150.01	855,475.15	862,602.44
	Earned profits	$\bar{\tilde{y}}_{2j}$	40,742.80	9,441.13	16,774.19	84,894.71	157,820.95

Table 2 presents the covariance of the inputs, outputs, and inputs with outputs used in models (2) and (3).

Next, we used the data presented in Table 2 to compute the results of the stochastic super efficiency model (2) considering $\alpha = 0.05$ for which $\varphi^{-1} = 1.645$ and $\alpha = 0.5$ for which $\varphi^{-1} = 0$. The stochastic super efficiency scores which are obtained using LINGO software for each DMU (bank branch) are presented in Table 3.

Table 2 Covariance of the inputs, outputs, and inputs with outputs

Covariance		*DMUs (bank branches)*				
		1	*1*	*1*	*1*	*1*
Inputs	$Cov(x_{1j}, x_{1j})$	45,518.75	84,256	84,761.75	23,220.25	159,580.75
	$Cov(x_{1j}, x_{2j})$	10,797.375	34,245	2,735.25	–1,174.125	–6,782.375
	$Cov(x_{2j}, x_{2j})$	15,429.75	27,372.5	15,547.25	14,853.25	8,113.25
Outputs	$Cov(y_{1j}, y_{1j})$	1,581	8,385	4,272.75	616.5	1,576.75
	$Cov(y_{1j}, y_{2j})$	–535.75	458.125	953	–72.375	681.5
	$Cov(y_{2j}, y_{2j})$	547.25	795.75	3,502.5	477.5	2,056.75
Inputs with outputs	$Cov(\tilde{y}_{1j}, \tilde{x}_{1j})$	–4,501.375	9,138.5	–6,896.375	1,915.875	–1,626.75
	$Cov(\tilde{y}_{1j}, \tilde{x}_{2j})$	–531	–3,142.75	1,298	413.125	–3,911.875
	$Cov(\tilde{y}_{2j}, \tilde{x}_{1j})$	–2,446.5	–3,177.5	–2,949.625	173.25	–819.875
	$Cov(\tilde{y}_{2j}, \tilde{x}_{2j})$	1,169	–2,029.25	–6,967	–480	–1,637.125

Table 3 Stochastic super efficiency scores

DMU	*α = 0.5* $\varphi^{-1}(\alpha) = 0$		*α = 0.05* $\varphi^{-1}(\alpha) = 1.645$	
	Efficiency	*Rank*	*Efficiency*	*Rank*
1	0.3755782	3	0.2748241	3
2	0.2525197	5	0.2005264	5
3	0.2693031	4	0.2695242	4
4	1	1	1	1
5	1	1	1	1

Now we can find the CSWs for the efficient DMUs (DMUs whose E scores are equal to one as presented in Table 3) according to model (3) (see Table 4) and calculate their efficiency scores by using equation (4) for two α scores (see Table 5).

Table 4 The weight scores of model (3)

Weight	*Score* *(α = 0.5)*	*Score* *(α = 0.05)*
v_1^*	$0.1 * 10^{-6}$	$0.1 * 10^{-5}$
v_2^*	$0.8414826 * 10^{-1}$	$0.8390680 * 10^{-1}$
u_1^*	$0.1140987 * 10^{-5}$	$0.1159283 * 10^{-5}$
u_2^*	$0.1 * 10^{-6}$	0

Table 5 Efficiency scores obtained using Table 8 and equation (4) for efficient DMUs

DMU	*Efficiency* ($\alpha = 0.5$)	*Efficiency* ($\alpha = 0.05$)
1	-	-
2	-	-
3	-	-
4	0.710	0.715
5	1	1

Finally, we can rank all the DMUs using Tables 3 and 5 (see Table 6).

Table 6 DMU rankings

DMU	*Ranks* ($\alpha = 0.5$)	*Ranks* ($\alpha = 0.05$)
1	3	3
2	5	5
3	4	4
4	2	2
5	1	1

The efficiencies of the five DMUs obtained by the five sets of weights are not comparable and therefore a CSWs using the ideal point method of model (3) were utilised. The efficiencies of the two efficient banks in model (3) are shown in Table 5 for two α scores. Table 6 shows the final ranking and it is evident that the proposed models can be used to generate common weights thus obtaining better results than the conventional DEA models. From the above numerical example, it can be seen that the proposed DEA model can successfully produce a complete ranking of the DMUs using the ideal point concept and a CSWs.

5 Conclusions and future research directions

Final ranking of the DMUs is an important phase in DEA. The conventional DEA methods usually rank the inefficient DMUs but do not provide sufficient information for ranking the efficient units. One of the commonly used methods for evaluating and ranking the efficient and inefficient DMUs is the CSW. In addition, the conventional DEA methods require precise input and output data while most real-life performance evaluation problems involve imprecise input and output data.

In this paper, we proposed a CSW for the SDEA model that uses the ideal point concept to deal with uncertainties inherent in the real-life performance evaluation problems. Using this method, we minimise the distance between the evaluated DMU and the ideal DMU and rank the DMUs to determine their input and output weights and obtain their efficiency scores. The optimal solution of the model proposed in this study was considered as a set of weights for all the DMUs used to rank the DMUs. We used a numerical bank efficiency example to demonstrate the applicability of the proposed method and exhibit the efficacy of the procedures. Future research can consider extending the model to problems with non-discretionary inputs and stochastic data.

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