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A bank merger predictive model using the Smoluchowski stochastic coagulation equation and reverse engineering

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Abstract

Purpose – The purpose of this paper is to provide a theoretical framework for predicting the next period financial behavior of bank mergers within a statistical-oriented setting.

Design/methodology/approach – Bank mergers are modeled combining a discrete variant of the Smoluchowski coagulation equation with a reverse engineering method. This new approach allows to compute the correct merging probability values via the construction and solution of a multi-variable matrix equation. The model is tested on real financial data relative to US banks collected from the National Information Centre.

Findings – Bank size distributions predicted by the proposed method are much more adherent to real data than those derived from the estimation method. The proposed method provides a valid alternative to estimation approaches while overcoming some of their typical drawbacks.

Research limitations/implications – Bank mergers are interpreted as stochastic processes focusing on two main parameters, that is, number of banks and asset size. Future research could expand the model analyzing the micro-dynamic taking place behind bank mergers. Furthermore, bank demerging and partial bank merging could be considered in order to complete and strengthen the proposed approach.

Practical implications – The implementation of the proposed method assists managers in making informed decisions regarding future merging actions and marketing strategies so as to maximize the benefits of merging actions while reducing the associated potential risks from both a financial and marketing viewpoint.

Originality/value – To the best of the authors' knowledge, this is the first study where bank merging is analyzed using a dynamic stochastic model and the merging probabilities are determined by a multi-variable matrix equation in place of an estimation procedure.

Keywords Asset size aggregation, Bank merger, Bank size distribution, Merging probability, Stochastic coagulation equation

Paper type Research paper

Introduction

Globalization has become a common vital aspect for the economic development of countries (Pushkin and Aref, 2004). Many countries are currently strengthening their competitive positions in the economic global market by upgrading their financial organizations. In order to be successful in the global economic market, large financial service firms often engage in mergers with other financial institutions (Berger *et al.*, 2000; DeYoung *et al.*, 2009). In spite of

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the fact that the mergers may succeed or fail, having higher assets by means of merging can sometimes be the only way to enter, stay, and compete in the global marketplace. In fact, corporate finance applies financial organization merging methods to use capital assets in the most appropriate way. In other words, if a country wants to introduce its economic and financial organizations to the world market, it must equip them with higher assets.

Mergers represent one way of increasing the asset size of an organization (DeLong, 2001; Ribeiro, 2010; Kolaric and Schiereck, 2014; Kim and Song, 2017). Banks are among the main financial organizations of a country and, hence, must unavoidably account for merging phenomena and market-related issues such as marketing efficiency (Piloff and Santomero, 1998), marketing costs and practice regulations (Dermine, 1999), acquisition of larger customer volumes vs the need for product rationalization (Harness and Marr, 2001), marketing integration processes and market-related performances (Homburg and Bucerius, 2005), strategic importance of branding and rebranding in the context of successive mergers and acquisitions (M&A) (Lambkin and Muzellec, 2008), and consumer-switching behavior (Al-Kwifi and Ahmed, 2015; Farah, 2017).

From the technical viewpoint, when two or more banks decide to merge their assets for a competitive advantage in the global environment, they face several practical difficulties and potentially critical risks. The basic practical questions are the following. Is merging possible? What is the probability of merging for banks with different asset sizes? How can this probability be modeled? And, how can the characteristics of the new organizations (i.e. the asset size) be defined?

In order to reduce the potential risks of the bank merger process, decision makers try to predict what will be the number of banks and their corresponding asset sizes in the next period.

This research suggests a theoretical framework for predicting the next year financial behavior by determining the best bank merger probability function and, consequently, evaluating the rates of change of the numbers of banks corresponding to different asset sizes.

The proposed bank merger model combines a discrete variant of the Smoluchowski coagulation equation with the reverse engineering method allowing for computing the correct merging probability values. To the best of our knowledge, this is the first study where bank merging is analyzed using a dynamic stochastic model and the merging probabilities are determined by a multi-variable matrix equation in place of an estimation procedure.

We follow the statistical approach proposed by Pushkin and Aref (2004), who describe bank mergers as scale-free coagulation processes. The reason for this is twofold. While the choice of implementing a statistical approach is motivated by the recent economic analysis of bank mergers, the use of the Smoluchowski coagulation equation reflects the necessity of designing a theoretical framework where the "physical" properties of bank markets are satisfied while guaranteeing reliable predictions on their development.

More precisely, Pushkin and Aref (2004) establish a bank merging model by introducing the probability $K(v, u)$ that two banks with assets of u and v million dollars merge during a unit time interval. This probability function describes the bank merger market since knowing its correct values, one can predict the evolution of the bank size distribution through the Smoluchowski coagulation equation.

In this study, we propose to model the bank merger process via a discrete variant of the Smoluchowski coagulation equation and develop a new procedure that allows to seek and find the correct values for the probability function $K(v, u)$.

We build on the version of the Smoluchowski equation implemented by Kaliviotis and Yianneskis (2009) in red blood cell (RBC) aggregation and blood viscosity modeling. As for the seeking procedure, it is based on a reverse engineering approach where the values for $K(v, u)$ are obtained through a multi-variable matrix equation. As a consequence, the proposed model not only provides a valid alternative to the standard estimation method approach but it also overcomes some drawbacks typical of estimation processes.

The proposed model is tested on real financial data collected from the National Information Center (NIC) website provided by the Federal Reserve System in American banks.

Finally, it must be noted that we interpret bank mergers as a stochastic process, whereas, in reality, such events only happen after complex and detailed planning and investigation. This fact represents a limitation of the proposed model since it cannot be used to analyze the micro-dynamic taking place behind the bank mergers. However, it has been shown that although most competitions between banks happen in local markets, the merger process is global. Two banks related to different geographic markets may never plan merging. In fact, they may not even know of each other's existence. Nevertheless, they participate in the global merger process by competing in the local markets that are not independent. This also explains why it is meaningful to taste the model considering the US banks as the totality of banks taking part in the merger process, and to refer to them as the bank merger market.

The bank merging phenomenon: a literature review

Bank significant elements and their implications in the economic sector have been widely studied (Llewellyn, 1998; Degryse *et al.*, 2008; Anginer *et al.*, 2014; Allen and Carletti, 2015).

Banks have developed many new innovations, such as bank merging, which has been adopted by countries across the world. There are numerous debates on the reasons why financial organizations such as banks should be merged as well as on the advantages and disadvantages associated with merging actions (Piloff and Santomero, 1998; Berger *et al.*, 2000; Becher, 2000; Focarelli *et al.*, 2002; Scholtens and de Wit, 2004; Koetter *et al.*, 2007; DeYoung *et al.*, 2009; Beccalli and Frantz, 2013; Hornstein and Nguyen, 2014; Ogura and Uchida, 2014; Du and Sim, 2016; Ogada *et al.*, 2016; Farah, 2017).

The main question in bank merging concerns merging efficiency. Is bank merging able to guarantee the desired profit? Considering the risks and the issues related to bank merging profitability, why are so many bank mergers taking place? Maximization of profit seems to be the main reason. Merging with other banks also allows a bank to increase its market share and take advantage of a new and expensive technology, thereby improving customer satisfaction (Federal Deposit Insurance Corporation, 1983; Kolaric and Schiereck, 2014).

Bank merging can be analyzed based on the four main regions where it has been taking place, that is, Canada, Asia, Europe, and the USA.

According to several researchers and practitioners who studied the Canadian banking sector, bank merging leads to further concentration of banking assets and financial power (Cooke, 2006). Unlike Canada, the USA has many mid-sized, small state, and local banks. Therefore, most Canadian banks tend to expand their activities by merging with smaller US banks. Unfortunately, the federal government does not foster bank merging and, consequently, bank merging is not considered as a profitable business sector in Canada (Cooke, 2006; Carroll and Klassen, 2010).

During the last decade, bank merging has occupied a major portion of the bank market of Asia. The merging phenomenon relates to the fact that banks are not as profitable as they were previously in that region. That is, most banks invest in asset merging as the best solution to avoid decreasing profits. However, due to the development and competitive environment of the bank market, bank merging in Asia is becoming more and more complex (Gilligan *et al.*, 2002; Hou *et al.*, 2012; Reddy *et al.*, 2013; Alam and Lee 2014; Healey and Chenying, 2017).

A sharp increase in bank merging has been registered also in the European Union. The successful experience of US banks encouraged European banks to consider bank merging to achieve high financial performance (Dymski, 2002; Altunbaş and Marqués, 2008; Heemskerk, 2013; Nnadi and Tanna, 2013, Kyriazopoulos and Drymbetas, 2015).

Despite the increasing tendency toward bank merging in Asia and Europe, the USA is the largest user of bank mergers. However, unlike other regions, US bank merging focuses on the domestic market rather than on the global one.

Berger *et al.* (2000) identified the removal of states in the US and federal restriction since the 1980s as the main reason for bank merging in the USA. According to Rhoades (2000), "From 1980 through 1998, about 8,000 bank mergers took place (equal to 55% of all banks existence in 1980), involving 2.4 trillion dollars in acquired assets." Dymski (2002) considered the cause and implication of global bank mergers in the US economy. Most studies indicate bank merging efficiency in the case of large US banks, and their results show that, since 1981, US banks with merging experience have become more profitable than other large banks.

In general, it can be concluded that mergers go hand in hand with the progress of business activities. In fact, transferring billions of dollars, managing thousands of employees and other properties can be only handled by financial mergers. At the same time, an analysis of dynamic mergers and their effects on the evolution of local markets shows that they are the strategic key to step up in the global market. That is, bank merging is one of the main strategies pushing local banks into the national bank system and national economic environment. Given the development of the bank market and the corresponding challenges, bank merging can be really considered a vital element for small banks to survive. Studies in this direction have been proposed by Wheelock and Wilson (2004), Reuben (2010), Finkelstein (2009), Akhigbe *et al.* (2017), and Mayorga Serna (2017), among others.

Finally, it is worth mentioning some marking issues immediately related to the M&A process. The implementation of a marketing perspective into the bank merging discussion is essential for the managers to successfully complete their challenging task.

One the most important marketing issues in successive M&A is how banking groups manage their branding (Lambkin and Muzellec, 2008). Every single merger involves branding or rebranding decisions, which affect the merging outcomes and lead to an update or change in the marketing strategies. Lambkin and Muzellec (2008) provide both a literature review on rebranding in the M&A context and an analysis of the key parameters to consider when deciding on which rebranding strategy to adopt.

More recent studies focusing on the marketing-related factors determining the value of corporate name changes include Kashmiri and Mahajan (2015) and Agnihotri and Bhattacharya (2017). In addition, mergers can be analyzed as a determinant of consumer-switching behavior, an extremely important factor in the retail banking sector, especially during financial crises (Brannen and Peterson, 2009; Kaur *et al.*, 2012; Al-Kwifi and Ahmed, 2015; Farah, 2017). Last but not least, mergers are a source of marketing information useful to examine the relationship between brand experience dimensions and brand familiarity with respect to financial services brands (Pinar *et al.*, 2012; Nysveen *et al.*, 2013; Levy and Hino 2016; Bapat, 2017).

Smoluchowski coagulation equations and the merging of financial organizations

The Smoluchowski coagulation equation was introduced by Smoluchowski (1916). This equation is typically used to define a system of partial differential equations modeling the diffusion and binary coagulation of a large collection of tiny particles. However, since aggregation and fragmentation are common natural phenomena, it allows for a wide variety of applications.

Through the years, there has been a considerable increase in the number of studies related to the different types of applications that the coagulation equation can have (Stockmayer, 1943; Drake, 1972; Pruppacher, 1978; Wall, 1980; Coveney, 1996; Lee *et al.* 2000; Begusova *et al.*, 2001; Mimouni and Wattis, 2009). All these applications are based on similar processes and common patterns. The main purpose is to simulate a dynamic model, which explains systems with different sizes of diffusion and high complexity. During the diffusion processes, particles join producing new particles with a larger size, while in the inverse processes big particles are broken into smaller ones. Therefore, a dynamic model is needed to explain all these changes.

Merging of financial organizations such as banks using the coagulation equation is one of the most recent applications of the coagulation equation.

By considering different studies in the bank merging literature, two main schemes can be recognized. The first scheme leads to a game-theoretical approach while the second is based on a statistical approach. In the first approach, the game theory is used to provide a description of the evolution of the banking system as a whole, including orders of magnitude in assets (Deneckere and Davidson, 1985; Gowrisankaran, 1999; Horn and Persson, 2001; Neary, 2003; Neary, 2007; Lukas *et al.*, 2012). On the other hand, the statistical approach follows the same microeconomic laws as the bank markets and, hence, the comprehensive macroeconomic description of the bank markets including dynamic characteristics (DeLong, 2001; Kampen, 2001; Cuesta and Orea, 2002; Valverde and Humphrey 2004; Al-Sharkas *et al.*, 2008).

Typically, the economic and financial studies following the statistical approach sketch a set of simple but acceptable rules for modeling micro-interactions of a financial system in such a way that the system follows physics rules and gives rise to power-law distributions. As a consequence, in the last two decades, the physics community has been very frequently involved in economic and financial projects where power-law distributions play a key role (Solomon and Richmond, 2001; Amaral *et al.*, 2001; Andriani and McKelvey, 2009; Simkin and Roychowdhury, 2011; Gleeson *et al.* 2014; Crawford *et al.*, 2015).

Bouchaud (2001) presents a short review of several power laws observed in economics and finance (e.g. in models of wealth distribution and price fluctuations) and discusses some physical models that could be useful to their development. A more recent review is provided by Gabaix (2009, 2016).

Several of the studies developed in the last two decades focus on risk models where risks are assumed to be heavy-tailed and on fitting results for sets of insurance loss data that are known to be heavy-tailed (Tang and Tsitsiashvili, 2003; Chen and Ng, 2007; Ahn *et al.*, 2012; Brazauskas and Kleefeld, 2016; Park and Kim, 2016).

Recently, dynamic power laws have been used as a tool for estimating the effect of progressive capital taxes on the distribution of wealth (Fernholz, 2017a). Furthermore, nonparametric econometric methods have been introduced to characterize general power-law distributions under basic stability conditions and allow for extensions in several social sciences directions (Fernholz, 2017b).

A stochastic tool that can be used to derive predictive models for power-low distributions is provided by the coagulation equations. Coagulation-fragmentation processes usually contain power-law tailed distributions. In particular, coagulation processes are suitable models for heavy-tailed distributions.

Since the data on large American banks present a power-law distribution (Pushkin and Aref, 2004), the Smoluchowski coagulation equation can be modified to analyze the bank merger process and formulate reliable predictions. In this sense, the analysis performed by Pushkin and Aref (2004) indicates that the Smoluchowski coagulation equation can provide a steady state for asset size distribution. In fact, the use of the Smoluchowski coagulation equation allows them to derive the coagulation model of bank mergers from economic data providing a framework totally different from the game-theoretical one.

Despite this significant connection between power-low distributions and coagulation equations, to the best of the authors' knowledge, the number of studies in the literature dealing with bank merger models based on coagulation equations remains limited.

Background models based on the Smoluchowski coagulation equation

The Smoluchowski coagulation equation is used to determine the rate of change of the particle size distribution of a given system. That is, for every possible particle size k, the

Smoluchowski coagulation equation describes the rate of change of the number density (i.e. number per unit volume) n_k of all the particles of size k due to coagulation.

As mentioned above, this study focuses on modeling the Smoluchowski coagulation equation so as to allow for reliable predictions regarding the evolution of financial data. The equations we build on are those proposed by Pushkin and Aref (2004) and Kaliviotis and Yianneskis (2009). These equations are outlined below.

When two banks are merged and a new bank forms as the result of the merger process, the total asset of the new bank is calculated by adding the assets of the two initial banks. Using the Smoluchowski dynamic equation, the average number of mergers creating banks of size v during a small time interval dt around time t is given by the following equation (Pushkin and Aref, 2004):

$$\frac{1}{2}\int_{v_0}^{v-v_0} K(v-u,u)n(t,v-u)n(t,u)du\ dt \tag{1}$$

where v_0 is the asset size of the smallest banks in merging action; $n(t, u)du$ is the average number of banks with total assets between u and $u+du$ million dollars at time t; $K(v-u, u)$ is the probability that banks with asset sizes $v-u$ and u million dollars merge.

On the other hand, the average number of mergers removing banks of asset size v during the same time interval dt is calculated as follows (Pushkin and Aref, 2004):

$$n(t,v)\int_{v_0}^{\infty} K(v,u)n(t,u)du\ dt \tag{2}$$

By dividing the change in $n(t, v)$ by dt, we obtain the following equation:

$$\frac{\partial n(t,v)}{\partial t} = \frac{1}{2}\int_{v_0}^{v-v_0} K(v-u,u)n(t,v-u)n(t,u)du - n(t,v)\int_{v_0}^{\infty} K(v,u)n(t,u)du \tag{3}$$

Equation (3) describes the rate of change of the number of banks $n(t, v)$ with asset size v in the case when the asset sizes are assumed to be continuous variables.

The discrete form of Equation (3) is as follows:

$$\frac{\partial n(t,v)}{\partial t} = \frac{1}{2}\sum_{u=v_0}^{v-v_0} K(v-u,u)n(t,v-u)n(t,u) - n(t,v)\sum_{u=v_0}^{\infty} K(v,u)n(t,\ u) \tag{4}$$

Another remarkable and very used variant of the Smoluchowski equation is the one used in RBC aggregation and blood viscosity modeling. This version considers both aggregation and fragmentation of merging particles (Kaliviotis and Yianneskis, 2009):

$$\frac{dn_k}{dt} = \frac{1}{2}\sum_{i=1}^{k-1} A(i,k-i)n_i n_{k-i} - \sum_{i=1}^{\infty} A(i,k)n_i n_k - \frac{1}{2}\sum_{i=1}^{k-1} D(i,k-i)n_k + \sum_{i=1}^{\infty} D(i,k)n_{k+i} \tag{5}$$

where dn_k/dt is the rate of change of the number of particles of size k due to coagulation; n_i is the number of particles of size i; n_k is the number of particles of size k; n_{k-i} is the number of particles of size $k-i$; n_{k+i} is the number of particles of size $k+i$; $A(i, k-i)$ is the probability that a particle of size i merges with a particle of size $k-i$; $A(i, k)$ is the probability that a particle of size i merges with a particle of size k; $D(i, k-i)$ is the probability that a particle is broken into two particles of size i and $k-i$; $D(i, k)$ is the probability that a particle is broken into two particles of size i and k.

By converting all the physical parameters into financial elements, Equation (5) extends Equation (4) to situations where the disaggregation of a bank in smaller ones is also allowed. This parameter conversion is described in the next section.

Proposed bank merger equation

In this section, all the physical parameters of Equation (5) are converted into financial elements so as to allow for financial applications. In particular, the banks in a merging market are regarded as biological particles that experience merging. In the biological approach, the key parameters to consider are the volume and size of the particles. The financial counterparts of these parameters are represented by the number of banks and their asset sizes.

The physical equation, Equation (5), gives the rate of change of the number of particles, n_k, of a certain size k due to coagulation. The main factors are the number of particles, n_k, as well as the probability of merging and demerging of these particles denoted by the $A(i, k)$ and $D(i, k)$, respectively. In the financial approach, the number of particles of a certain size is interpreted as number of banks with a certain asset size, while the merging and demerging probabilities relative to two particles correspond to the merging and demerging probabilities of two banks.

Thus, Equation (5) yields the following new formulation for the coagulation equation that represents the proposed bank merger equation:

$$\frac{dn_v}{dt} = \frac{1}{2}\sum_{u=v_0}^{v-v_0} K(v-u, u)n_{v-u}n_u - \sum_{u=v_0}^{\infty} K(v, u)n_u n_v$$

$$-\frac{1}{2}\sum_{u=v_0}^{v-v_0} L(v-u, u)n_u + \sum_{u=v_0}^{\infty} L(v, u)n_v \tag{6}$$

where dn_v/dt is the rate of change of the number of banks, n_v, with asset size v due to merging; v_0 is the asset size of the smallest banks in merging action; n_{v-u} is the number of banks with asset size $v-u$; n_u is the number of banks with asset size u; n_v is the number of banks with asset size v; $K(v-u, u)$ is the probability that banks with asset size $v-u$ are merged with banks with asset size u; $K(v, u)$ is the probability that banks with asset size v are merged with banks with asset size u; $L(v-u, u)$ is the probability that a bank is demerged into two banks with asset size $v-u$ and u; $L(v, u)$ is the probability that a bank is demerged into two banks with asset size v and u.

Equation (6) completes Equation (4) by contemplating the possibility that also demerging actions take place in the market.

There are many reasons to conclude that bank mergers have the same properties as physical particle systems and confirm that a statistical physics approach is appropriate. First, bank merging is not limited to banks of a certain size. In other words, bank merging covers all banks with any asset size. Second, bank merging is not limited to the particular market or region. Although banks experience different market environment in different situations and regions, the merging process happens for all of them in the same way and following the same pattern. Third, the overall dynamic of the bank market is only weakly dependent on the particular bank interactions.

Implementing of the bank merger equation

With respect to the new developed equation, there are two main elements that should be defined in order to implement the model. The first element is the bank size distribution

$n(t, u)$, where $u \geqslant v_0$, at a generic time t. This can be determined by using the following two methods:

(1) the least square regression and curve fitting; and

(2) the Zipf and Hill plotting.

The results of the previous research are mostly based on the second method and the incorporation of power-law distributions.

The second element is represented by the aggregation and disaggregation probability values, i.e., $K(v-u, u)$, $K(v, u)$, $L(v, u)$, and $L(v-u, u)$. Clearly, this fact poses the problem of applying a procedure able to assign the probability values so as to reflect the real probability distribution at the generic time t.

Once the aggregation and disaggregation probability values (i.e. $K(v-u, u)$, $K(v, u)$, $L(v, u)$, and $L(v-u, u)$) and the bank size distribution values (i.e. n_{v-u}, n_u, and n_v) have been determined, Equation (6) can be used to compute the rate of change of the number of banks with a given asset size in the current year.

Finally, the number of banks with a given asset size can be predicted for the next year using the following equation:

$$V^{next} = \left| V^{current} + \left(V^{current} \cdot \text{Rate of change of } V \right) \right| \tag{7}$$

where V is the number of banks n_u with a certain asset size u, V^{next} is the value of V predicated for the next year, and $V^{current}$ is the current year value of V.

Determining the merging probability values

There are two methods that can be used to seek and find the appropriate probability values. The first method checks different constant probability values, determines the rate of change and, consequently, predicts the next year value. It then plots the bank size distribution and tests to see if it has a power-law distribution. The second method follows a reverse engineering approach. In this approach, the data relative to the bank size distributions of different years are gathered and used to extract the corresponding rates of change. The numbers of banks with different asset sizes are also extracted from the current data. Finally, the probability values are calculated exactly by solving a multi-variable equation.

Estimation method and drawbacks

Using the estimation method, the probability functions are selected randomly during different iterations, and the best estimate is used to calculate the rate of change of the number of banks with a certain size. More precisely, a generic MATLAB code is designed on the basis of Equation (6) and its outcome is the rate of change relative to the current year of the number of banks in function of their asset sizes. This also allows to estimate and plot the number of banks against the asset sizes. Finally, the rate of change returned by implementing the coagulation equation with the estimated probability values is used to construct the bank size distribution for the next year.

The main assumptions of this approach are as follows:

(1) For every $u \geqslant v_0$, $K(v-u, u)+K(v, u)+L(v, u)+L(v-u, u) = 1$.

(2) For every $u \geqslant v_0$, $K \geqslant L$ (i.e. at the first iteration there is no demerging, just merging).

The results obtained implementing the estimation method to analyze real data (see the "Estimation Method Results" section below) show that, although the probability values randomly determined by the MATLAB code are able to provide the desired trend for the asset size distribution, they do not suffice to compute accurate rates of change. In other words, randomly generated probabilities can predict trend and shape of a distribution for the next year, but they cannot predict accurate numerical values for the distribution parameters.

Proposed reverse engineering method

In order to use this method, Equation (6) needs to be adapted to allow for the definition of a solvable matrix equation where the number of unknowns (the merging/demerging probability values) equals to the number of underlying equations (these equations are based on the bank and asset size distributions).

The matrix equation that must be constructed from Equation (6) is the following:

$$\underbrace{\begin{bmatrix} \square & \cdots & \square \\ \vdots & \ddots & \vdots \\ \square & \cdots & \square \end{bmatrix}_{m\times m}}_{A} \times \underbrace{\begin{bmatrix} \square \\ \cdots \\ \square \end{bmatrix}_{m\times 1}}_{X} = \underbrace{\begin{bmatrix} \square \\ \cdots \\ \square \end{bmatrix}_{m\times 1}}_{B} \tag{8}$$

where A is a square matrix whose entries are the products $(n_u \cdot n_v)$ or $(n_{v-u} \cdot n_v)$ defining Equation (6) at time t (i.e. for the year t); A^{-1} is the inverse matrix of matrix A; X is a column vector whose entries are the merging/demerging probability values $K(v-u,u)$, $K(v,u)$, $L(v,u)$, $L(v-u,u)$; B is a column vector whose entries are the rates of change dn_v/dt relative to different times t (i.e. different years).

The entries of matrix A are known values since they can be computed using the current data collection. If the entries of B were also known values, then the only unknown values would be the probability values composing X and solving the matrix equation would yield the exact values of the merging probabilities.

Thus, we propose the following rule to extract the rates of change dn_v/dt composing B from the data collection:

$$\begin{aligned} \frac{dn_v}{dt} &= -\frac{1}{2}\left[\text{Average}((\text{Ratio of } (\log_{10}(\text{Number of banks}))-1))\right] \\ &= -\frac{1}{2}\frac{\sum_{u=v_0}^{v}((\text{Ratio of } \log_{10}(\text{Number of banks}))-1)}{v} \end{aligned} \tag{9}$$

Using Equation (9), X becomes the unknown of the matrix equation, Equation (8), whose solution, as anticipated above, provide the exact values of the merging probabilities. The solution method is well-known in matrix algebra:

$$\begin{aligned} & A \times X = B \\ & A^{-1} \times A \times X = A^{-1} \times B \\ & I \times X = A^{-1} \times B \\ & X = A^{-1} \times B \end{aligned} \tag{10}$$

Model development with real data

In this section, we apply the proposed bank merger model to real data and discuss the results obtained by implementing both the estimation and the reverse engineering approaches. In particular, our analysis shows that the behavior of the bank size distributions predicted by the reverse engineering approach is much more adherent to the actual financial data than that derived from the estimation method.

As a consequence, the proposed reverse engineering method turns out to be a suitable tool that can help to maximize the benefits of merging business and financial organizations while reducing the potential risks associated to it.

Step 1: data collection

In this research, the financial data are extracted from the NIC website[1] provided by the Federal Reserve System in American banks. The financial data are summarized in BHCPR Peer reports for each year quarter. Since the asset sizes of the single banks belong to a very wide range, they are divided into six main categories as shown in Table I.

The main factors to retrieve from financial reports for the model to be developed are two: the number of banks in each peer so that the number of banks with a certain asset size is obtained; and the total asset size of each peer. In order to obtain the total asset size of a peer, the asset sizes of all the banks belonging to the same peer's category are extracted and added together.

Step 2: data processing

The number of the banks in each peer and the corresponding asset sizes relative to the last quarter of each year from 2007 to 2015 is transferred to Excel Spreadsheet for further calculation. The data collection relative to the years 2007-2015 is sorted out and four values isolated for each year. As explained above, the first two values must be the number of banks in each peer and the total asset size of each peer.

The other two values needed for analyzing the bank merger market are obtained by computing the logarithm in base 10 of both the number of banks and the total asset size of each peer. Finally, for a better data analysis, all these values are summarized in separate tables, one per each year. As an example, Table II illustrates all the information relative to the year 2010.

Step 3: data implementation for the estimation method

In this section, the MATLAB code is designed. The input data are the values obtained in the previous section. Then, the MATLAB code is run for the input data. Finally, the results are plotted and used to evaluate further results.

Peer No.	Asset size range
1	\$10B and over
2	\$3B-\$10B
3	\$1B-\$3B
4	\$5,000MM-\$1B
5	Less than \$500 million
6	Typical 2nd tier

Table I. Bank asset sizes divided into six peers

Peer No.	No. of banks	Total asset size	Log_{10} (No. of banks)	Log_{10} (Total asset size)
1	70	12,803,847,764	1.85	10.11
2	88	494,542,378	1.94	8.70
3	297	493,324,908	2.47	8.69
4	438	311,190,432	2.64	8.49
5	91	34,969,978	1.96	7.54
6	78	2,060,739,496	1.90	9.31

Table II. Data collection summary for year 2010

The MATLAB code is divided into two main parts: the first part contains the assumptions and generates random numbers for the merging probabilities of Equation (6); and the second part not only reads the values generated randomly from the first part, but it also reads the data processed for the single year, that is, the number of banks in each peer and the corresponding total asset size. Consequently, Equation (6) is applied.

Figure 1 shows the merging probability values randomly generated by MATLAB according to the estimation method.

Step 4: data implementation for the reverse engineering method

For a correct implementation of the processed data in Equation (8), we need to use Equation (5) with the parameters appropriately interpreted for a financial viewpoint. Indeed, Equation (5) reflects the choice of dividing banks and assets among six peers better than Equation (6).

Therefore, to test the proposed reverse engineering method in the case of six peers, we use the following variant of the Smoluchowski coagulation equation:

$$\frac{dn_k}{dt} = \frac{1}{2}\sum_{i}^{k-i} A(i, k-i)n_i n_k - \sum_{i}^{k} A(i, k)n_i n_k; \qquad i = \{1, 2, \ldots, 6\}, \quad k = 6 \tag{11}$$

which yields the following:

$$\frac{dn_k}{dt} = A(1,5)n_1n_5 + A(2,4)n_2n_4 + A(3,3)n_3n_3 * 0.5 - A(1,6)n_1n_6 - A(2,6)n_2n_6$$
$$- A(3,6)n_3n_6 - A(4,6)n_4n_6 - A(5,6)n_5n_6 - A(6,6)n_6n_6 \tag{12}$$

Using Equation (12), the processed data of Step 2 (Data processing) are used to create nine equations, one per each year in the range 2007-2015. Therefore, a system of nine equations in

Columns 1 through 7

0.0945	0.2114	0.0455	0.1616	0.2249	0.3329	0.1146
0.2370	0.1672	0.2087	0.3070	0.0118	0.2352	0.2766
0.0489	0.1895	0.3640	0.2313	0.2406	0.1085	0.1308
0.3766	0.2208	0.7563	0.2080	0.2933	0.2100	0.0200
0.2908	0.3239	0.2494	0.3082	0.1761	0.2561	0.1714
0.5272	0.7811	0.2559	0.3211	0.0969	0.5936	0.0699
0.3701	0.4075	0.3443	0.2198	0.4513	0.7223	0.1465
0.2667	0.2695	0.0736	0.2698	0.3249	0.4001	0.6311
0.3965	0.2797	0.8032	0.3260	0.2462	0.8319	0.8593
0.2968	0.1116	0.2753	0.4564	0.3427	0.1343	0.9742
0.4448	0.8997	0.7167	0.7138	0.3757	0.0605	0.5708
0.9880	0.4504	0.2834	0.8844	0.5466	0.0842	0.9969

Columns 8 through 12

0.1982	0.2325	0.1631	0.1594	0.4563
0.2535	0.4248	0.1856	0.5778	0.0720
0.1602	0.2030	0.5048	0.4687	0.1951
0.1033	0.1468	0.3121	0.6327	0.3148
0.4443	0.1384	0.3126	0.3626	0.3058
0.0691	0.4056	0.3454	0.3254	0.0404
0.5072	0.3418	0.2155	0.3254	0.1586
0.0646	0.3151	0.2900	0.4534	0.4570
0.4362	0.6128	0.3596	0.2361	0.0173
0.8266	0.8194	0.1565	0.5399	0.6663
0.3945	0.5319	0.5621	0.9452	0.0471
0.6135	0.2021	0.6948	0.7842	0.6690

Figure 1.
Aggregation probability values

nine unknowns is obtained. This system can be rewritten as a matrix equation of the form displayed by Equation (8).

At the same time, using Equation (9), the data collected per each year (i.e. Table II) are used to calculate the rate of change of the number of banks with a certain size.

The solution obtained for the multi-variable equation (Equation (8)), that is, the merging probability values (i.e. $A(i, k-i)$, $A(i, k)$) determined by implementing the processed data in the proposed reverse engineering approach, are displayed in first two column of Table III. The details relative to the construction of the multi-variable equation and the solving procedure applied to obtain these results are available in Appendix 1.

Table III also compares the solution provided by the reverse engineering approach with that obtained by implementing an improved reverse engineering approach. The improved approach is discussed in the following section where an analysis of the results is performed and the accuracy and feasibility of the model is tested against its ability to predict the next year behavior of the bank mergers.

Data analysis and results

Estimation method results

Using the estimation method, the merging probability values are obtained iteratively using MATLAB and their best estimation used to calculate the rate of change of the number of banks with a certain size. Hence, considering this rate, the bank asset size distribution of the next year can be predicted.

The initial results are plotted as both a bar chart and a line graph in Figures 2 and 3, respectively. The black boxes shown in Figure 2 represent the 2006 results, while the red boxes represent the 2007 data.

The bar chart shows only black boxes in segments 1 and 7, which indicate the rate of change is negative at these two points and, consequently, there is no red box. Apart from this, the rates of change in the number of banks, which are the differences between the black and red boxes, are very high. They are higher than the actual values, which means that randomly generated probabilities are not accurate enough. They just have the ability to track the pattern.

In other words, although a random probability distribution can plot the desired trend regarding the basic formula, it does not suffice to calculate accurate changes. Random probabilities can predict trend and shape of a distribution for the next year, but they cannot predict accurate numerical values for the distribution parameters.

The line graph shows that plotting the logarithms in base 10 of the numbers of banks vs those of the asset sizes yields a power-law distribution. In addition, it can be observed that, after the sharp decrease taking place in the first segment, there is a steady behavior till the end of period.

Reverse engineering method		Improved reverse engineering method	
$A(i, k-i)$, $A(i, k)$	Aggregation probabilities	$A(i, k-i)$, $A(i, k)$	Aggregation probabilities
$A(1, 5)$	6.47753E−05	$A(1, 4)$	1.24137E−06
$A(2, 4)$	1.82665E−05	$A(2, 3)$	2.60257E−05
$A(3, 3)$	−2.78223E−05	$A(1, 5)$	3.60416E−06
$A(1, 6)$	0.000429496	$A(2, 5)$	0.000103873
$A(2, 6)$	0.000301283	$A(3, 5)$	0.000151443
$A(3, 6)$	−0.000455575	$A(4, 5)$	2.53903E−05
$A(4, 6)$	0.000112501	$A(5, 5)$	0.000648947
$A(5, 6)$	0.000218185		
$A(6, 6)$	0.000213908		

Table III. Aggregation probabilities: reverse engineering vs improved reverse engineering

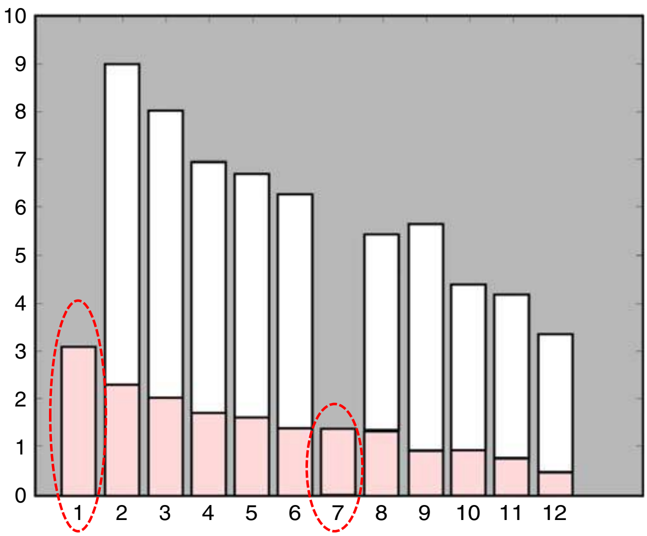


Figure 2.
Bar chart result for 2006 and 2007 data

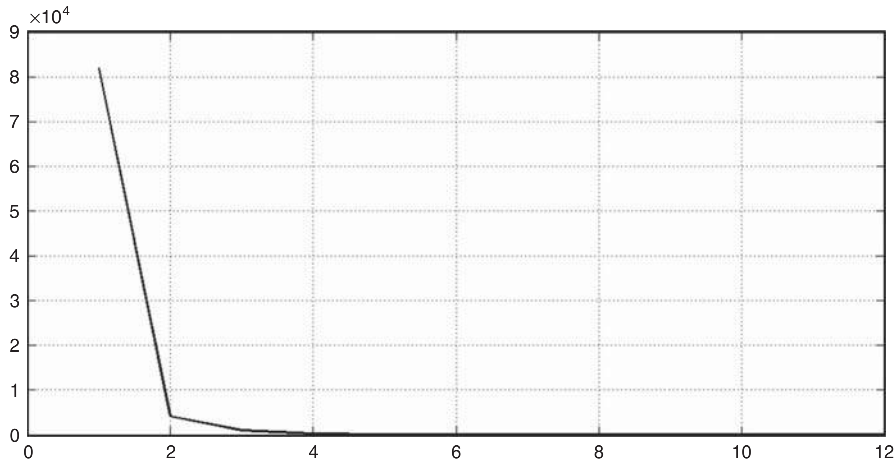
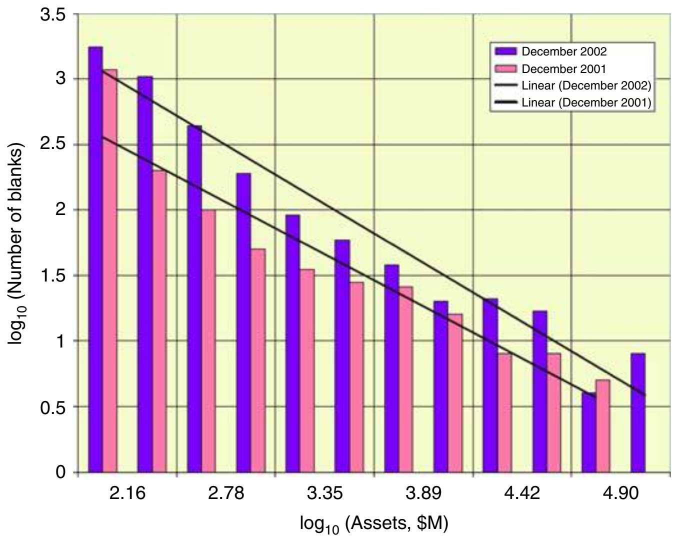


Figure 3.
Line graph result for 2006 and 2007 data

This sharp decrease indicates that the number of banks with a small asset size is greater than the number of banks with a high asset size in the USA. This trend can be used to explain the increasing wave of bank mergers in the USA. The existence of many banks with small asset size would be the first and main reason for the bank merging phenomenon to be so popular in the USA, compared to other regions.

Finally, a comparison between the bar chart and the actual data distribution shows approximately the same trend. This being the case, it can be concluded that bank merging undergoes to processes that can be assimilated to those of a physical system modeled through the Smoluchowski coagulation equation. Both bank merging and the coagulation process change with an exponential trend. In addition, by comparing Figures 2-4, it can be concluded that the numbers of banks as well as the bank asset sizes have a power-law distribution.



Source: Pushkin and Aref (2004)

Figure 4. Histogram of bank assets for 2001 and 2002

Reverse engineering method results

In this section, we discuss the results obtained for the merging probability values in the six-peer case (first and second column of Table III) and compare them with those derived in a five-peer case (third and fourth column of Table III).

As shown in Table III, two of the nine merging probabilities determined in the six-peer case, i.e. $A(3, 3)$ and $A(3, 6)$, take a negative value. This fact is clearly a structural problem since probability values ought to be both positive and less than 1.

In the Smoluchowski coagulation equation, the number of aggregation probabilities changes with the number of particles. That is, the number of unknowns in the multi-variable equation varies with the number of peers used to process the current data collection. Consequently, while operating with six peers produces negative probability values, assuming a different number of peers can suffice to overcome the problem. We show that this is the case by analyzing the data in the five-peer case.

Improving the reverse engineering method by reducing the number of peers

In the six-peer case, Equation (12) is obtained from Equation (11) in order to compute the rate of change of the number of banks with a certain asset size relative to the generic year t.

Suppose now that the peers become five. Then, Equation (11) yields a different expression for the computation of the rate of change for year t, that is:

$$\frac{dn_k}{dt} = \frac{1}{2}[A(1,4)n_1n_4 + A(2,3)n_2n_3 + A(3,2)n_3n_2 + A(4,1)n_4n_1]$$

$$-[A(1,5)n_1n_5 + A(2,5)n_2n_5 + A(3,5)n_3n_5 + A(4,5)n_4n_5 + A(5,5)n_5n_5]$$

which, after some algebra operations, becomes:

$$\frac{dn_k}{dt} = A(1,4)n_1n_4 + A(2,3)n_2n_3 - A(1,5)n_1n_5 - A(2,5)n_2n_5 - A(3,5)n_3n_5$$

$$-A(4,5)n_4n_5 - A(5,5)n_5n_5 \tag{13}$$

A quick comparison of Equations (12) and (13) shows that the negative probabilities disappear when reducing the number of peers from six to five. More in general, when the number of groups changes from an even to an odd number, all the probabilities are positive.

After removing peer 6, the matrix equation in Equation (8) must be constructed for the five-peer case (see Appendix 1). The merging probability values obtained by solving the corresponding multi-variable equation are reported in last two columns of Table III and are all positive. Hence, the improved reverse engineering approach allows to overcome the difficulty of having to deal with negative probability values.

Predicting the next year values

Once the merging probabilities have been determined, they can be used to predict the next year financial behavior. Since the data implemented cover the year range 2007-2015, we make predictions for the year 2016. Finally, a comparison of the predicted data with the real ones from 2016, which are already available, will prove the accuracy of the proposed model.

The merging probability values obtained in the improved five-peer case (see third and fourth column of Table III) are implemented in Equation (13) together with the real values of n_k ($k = 1, \ldots, 5$) from the year 2015. Hence, Equation (13) is solved for dn_k/dt.

For example, the rate of change obtained from these calculations for the number of banks in peer 5 is equal to −2.11138 percent.

Finally, these rates of change are used to predict the bank size distributions for the year 2016 based on those known from the year 2015.

These predictions are made applying Equation (7) with the logarithms in base 10 of the number of banks. More precisely, we use the following:

$$\log_{10}\left(n_k^{in2016}\right) = \left|\log_{10}\left(n_k^{in2015}\right) + \left(\log_{10}\left(n_k^{in2015}\right)\cdot\frac{dn_k}{dt}\right)\right| \tag{14}$$

where: n_k is the number of banks with asset size k ($k = 1, \ldots, 5$), n_k^{in2016} is the value of n_k predicated for the next year, n_k^{in2015} is the current value of n_k.

The values predicted for the number of banks n_k and the corresponding asset size k (where $k = 1, \ldots, 5$) for the year 2016 using the merging probabilities provided by the improved reverse engineering method and Equation (13) are summarized in Table IV. Table IV also shows the actual values of these numbers for the year 2016, which are already available in the website (see footnote 1).

A comparison of the predicted values with the actual ones relative to the year 2016 allows to check the reliability of the proposed model. Indeed, Table IV shows that the predicted and actual values for the number of banks in each peer differ less than 10 percent from one another.

Table IV.
Predicted values vs actual values for year 2016

Peer No.	Actual values		Predicted values	
	No. of Banks	Log_{10} (No. of Banks)	No. of Banks	Log_{10} (No. of Banks)
1	100	2	106	2.03
2	150	2.18	226	2.39
3	325	2.51	357	2.63
4	41	1.61	91	1.96
5	5	0.69	7	0.84

The precision in the values predicted for the next year show that of the proposed financial Smoluchowski model can be used to actually predict the number of banks corresponding to a certain asset and how it will be varying (its trend) in the future years.

Managerial implications

From a managerial viewpoint, the aggregation probability values provided by the proposed reverse engineering approach (Table III) as well as the adherence to the actual financial data shown by the predictions obtained for the next year (Table IV) have immediate and interesting consequences in terms of future bank activities and strategies.

The implementation of the proposed approach allows managers to take informed decisions regarding merging actions and marketing strategies with an important advantage with respect to the use of standard estimation methods. On the basis of the results obtained in the previous sections, the following managerial considerations can be outlined and used as a practical guideline in similar contexts.

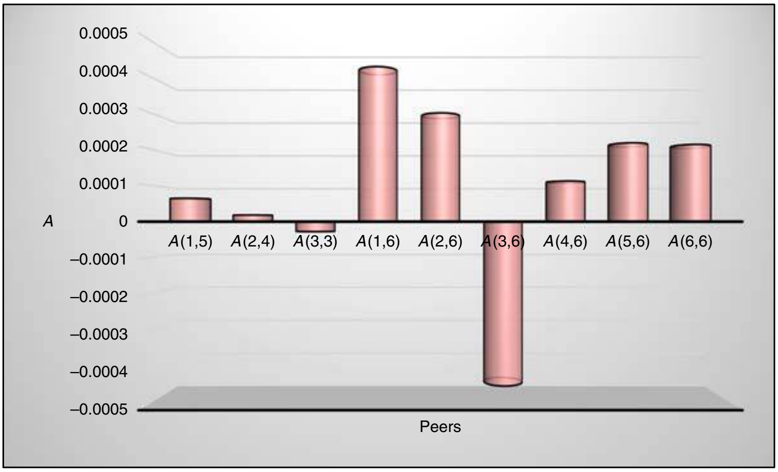
An analysis of the aggregation probability values obtained in the six-peer and five-peer cases reveals the merging trend allowing for a comparative discussion of the possible outcomes. To facilitate this analysis, a graphical comparison among the merging probabilities obtained in the six-peer and five-peer cases is provided by Figure 5:

- As shown in Figure 5(a), in the six-peer case, the highest probability value is given by $A(1, 6)$. This means that the banks in peer 6 have a higher probability to be merged with banks belonging to peer 1 than to be merged with those in other peers. Peers 1 and 6 also show the smallest difference in the numbers of banks and asset sizes. Thus, for small size banks, the probability of merging with like ones is higher than that of merging with big ones. In addition, peer 6 contains banks that have experienced merging during the previous year, witnessing the fact that banks that have already merged are potentially more prompt to be merged again. On the other hand, the lowest probability value is given by $A(2, 4)$, which illustrates the fact that banks presenting the highest difference in asset size are also the less probable to be merged. In other words, the closer the asset sizes of two banks are the higher their merging probability is.
- As shown in Figure 5(b), in the improved five-peer based model, the highest merging probability value corresponds to banks within the same group, that is, with the same asset size. This value is $A(5, 5)$. Therefore, the improved model is more accurate than the original approach. The original approach shows that the smaller the difference in asset sizes, the higher the merging possibilities. The improved model shows a better result, that is, the banks with the highest merging possibilities are those whose asset sizes belong to the same peer.

Further implications regarding the merging trends can be inferred from the results displayed in Table IV:

- Tracing back the merging trends and numbers of banks over the last ten years (2007-2016), it is evident that peer 5 is the one showing the highest initial rate of change (in year 2007). As the years go by, the number of banks in peer 5 reduces significantly (i.e. they experience more merging) and the rate of change shifts from a positive value to a negative one (−2.11138 percent) for 2016 (i.e. at the end of 2015). The negative rate of change registered for peer 5 at the end of 2015 vs the positive rate of change registered at the beginning of the ten-year observation period reflects the decrease of the number of merging actions performed by peer 5 over consecutive years. This is due to an increase in the asset size of the banks in peer 5. That is, through the years, the banks in peer 5 have formed mega banks which explains the negative rate of change for peer 5.

(a)



(b)

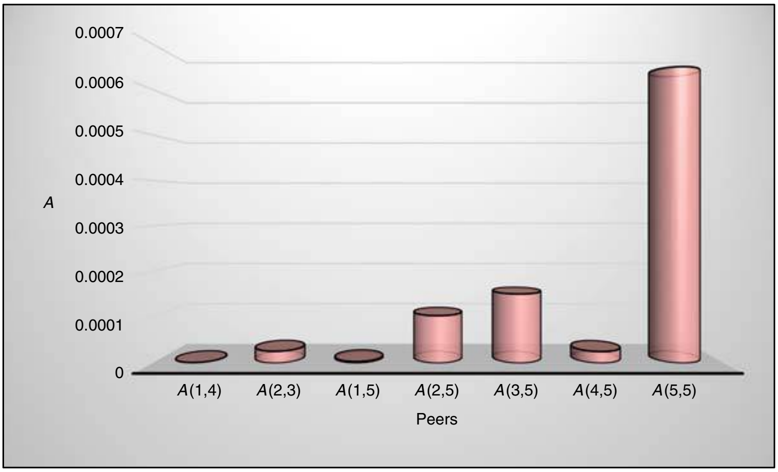


Figure 5.
Reverse engineering model vs improved reverse engineering model

Notes: (a) Aggregation probability values in the six-peer case; (b) aggregation probability values in the five-peer case

- On the other hand, the numbers of banks related to the other peers, highlighted with a red circle in Table IV, increase significantly indicating the lack of incentives for the peers of mega banks to merge with banks with a smaller asset size.

The points discussed above concern the economic and financial settings. However, bank merging is a challenging task affecting different aspects of management. Another important set of implications that must be taken into consideration when analyzing M&A past decisions in view of new ones relates to marketing.

One the most important marketing issues managers must deal with in successive M&A processes is how banking groups manage their branding (Lambkin and Muzellec, 2008). Branding problems depend on several factors such as the size and international statue of the banks participating in the merging process. Every single merger involves branding or rebranding decision, which affects the outcomes of merging and the subsequent market-related performances.

The key parameters that should be considered in branding and rebranding strategies include relative sizes of the companies that will be merged, type of services/products, correlation between markets and products, and finally the geographical distances among mergers (Berger *et al.*, 1999; Kaplan, 2006; Lambkin and Muzellec, 2008). With respect to the first set of parameters, it has been observed that organizations smaller in size and, hence, less strong on the market, tend to be rebranded. The weaker the merger, the higher the need to look for a new position in the market. On the other hand, large organizations undergo to minimum changes regarding branding strategies (Lambkin and Muzellec, 2008).

Given that the relative asset sizes of the merging banks are parameters common to both financial and marketing contexts, and that, in this study, asset size distributions constitute the main validation tool for the merging probabilities, our model also provides a decision-making schema for the branding and rebranding of the new entities after merging.

For instance, according to our results, there is a higher probability that small size banks are merged with like ones rather than with big ones. Then, a joint brand would be the best option. A joint brand for merging banks indicates a merger of equals, where each bank comes back to its own customers stronger (Basu, 2006).

We have also observed that banks whose asset sizes differ the most are the most unlikely to be merged. Then, should two such banks merge, the branding policy should be different. In fact, in this case, a unique brand name corresponding to the one of the acquirer (i.e. the bank with the higher asset size) would be the best option (Basu, 2006).

Conclusion and recommended studies

The main purpose of this study has been to identify a dynamic stochastic model for the bank merger process. Following the large literature existing on bank mergers and, in particular, the statistically oriented research on the topic, we have proposed a model based on the Smoluchowski coagulation equation.

Following the statistical approach to bank merging introduced by Pushkin and Aref (2004), we have designed a theoretical framework where bank markets are interpreted and analyzed as physical systems.

The use of the Smoluchowski coagulation equation has allowed to derive a coagulation model of bank mergers where the next year financial behavior can be predicted on the basis of economic data. This has yielded a model totally different from those typically developed in the game-theoretical setting.

The proposed bank merger model combines a discrete variant of the Smoluchowski coagulation equation with the reverse engineering approach. As a result, a new procedure has been developed that allows to seek and find the correct values for the merging probability values through the construction and solution of a multi-variable matrix equation.

We have tested the proposed model on real financial data from the NIC website provided by the Federal Reserve System in American banks.

We have discussed the results obtained by implementing both the estimation and the reverse engineering approaches. In particular, our analysis shows that the behavior of the bank size distributions predicted by the reverse engineering approach is much more adherent to the actual financial data than that derived from the estimation method.

Therefore, the newly introduced combination of the Smoluchowski coagulation equation with the reverse engineering approach not only provides a valid alternative to the standard estimation method approach but it also allows to overcome some drawbacks typical of estimation processes.

Since the real data implemented in the model covered 2007-2015, we have made predictions for the year 2016 and compared them with the real ones. The results have shown that there is no significant gap between the predicted and the actual data for the next year, this fact proving the higher accuracy of the proposed model with respect to any single approach.

Another achievement of this research relates to the role of the rate of change of the number of banks during a certain period. The rate of change of the number of banks can be a positive or negative value. As long as the data show a positive rate, merging will be taking place. The rate of change becoming a small negative value indicates the existence of mega banks in some peers and an increase in the number of banks in other peers. Finally, if the rate of change takes a high negative value and the numbers of banks in all peers are small, there is a significant number of mega banks. At this point, the disaggregation part should be added to the model and the disaggregation probabilities of banks should be analyzed. Appendix 2 provides a comparison of the positive rates of change during the first ten-year range, i.e., 2003 to 2012, vs the negative rate of change during the second ten-year range, from 2007 to 2016.

One limitation of this study is the following. Although banks as a financial organization have several important parameters, this research considers only two main parameters for bank merging, that is, the number of banks and asset size. Further studies should comprise the evaluation of other factors (i.e. staff and/or properties) as part of the bank merger process.

Another shortcoming of this research is that it does not cover bank demerging. The main reason is that there is no financial data or approach considering bank demerging. However, in the future, bank demerging might be more profitable than bank merging. Thus, a new model with both aggregation and fragmentation parts should be developed.

The following recommendations are provided in order to improve the proposed model and the analysis of the bank merging phenomenon:

- In this paper, the US bank merging is considered as a case study. However, all banks over the world could be considered as the case study of this model, and the results of different regions compared.
- Bank demerging might also be considered as a useful financial approach for future research. An extension of the model could be developed with both bank merging and demerging aspects.
- New assumptions could be added to avoid the occurrence of negative probabilities in the bank merger process.
- Partial bank merging is also a new idea that could be implemented in this model. That is, it could be assumed that two banks merge but they do not share their entire assets. They merge only the asset size necessary for the merging to be profitable for them.
- Predictions relative to asset sizes could be considered as an improvement option. A new model could be designed so as to predict the number of banks and new asset size after merging in the next year.

Regarding the last point, future studies can be developed focusing on mega banks and balance assets aiming at the design of a model able to systematically identify mega banks and analyze their behavior.

Note

1. www.ffiec.gov/nicpubweb/nicweb/nichome.aspx

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Appendix 1. Data implementation for the reverse engineering method: years 2007-2015

Construction of the multi-variable equation

The starting point is Equation (11), recopied and relabeled below:

$$\frac{dn_k}{dt} = \frac{1}{2}\sum_{i}^{k-i} A(i,k-i)n_i n_k - \sum_{i}^{k} A(i,k)n_i n_k; \quad i = \{1,2, \ldots, 6\}, \quad k = 6 \tag{A1}$$

Expanding the equation, we obtain:

$$\frac{dn_k}{dt} = \frac{1}{2}[A(1,5)n_1n_5 + A(2,4)n_2n_4 + A(3,3)n_3n_3 + A(4,2)n_4n_2 + A(5,1)n_5n_1]$$

$$-[A(1,6)n_1n_6 + A(2,6)n_2n_6 + A(3,6)n_3n_6 + A(4,6)n_4n_6 + A(5,6)n_5n_6 + A(6,6)n_6n_6]$$

Since $A(1,5) = A(5,1)$ and $A(2,4) = A(4,2)$, after some algebra operations, the above equation becomes:

$$\frac{dn_k}{dt} = A(1,5)n_1n_5 + A(2,4)n_2n_4 + A(3,3)n_3n_3{*}0.5 - A(1,6)n_1n_6 - A(2,6)n_2n_6$$

$$-A(3,6)n_3n_6 - A(4,6)n_4n_6 - A(5,6)n_5n_6 - A(6,6)n_6n_6 \tag{A2}$$

which is implemented for each year in the range 2007-2015. The equation corresponding to the year 2010 is reported below as an example. That is, using the data collected in Table II, we have:

$$\frac{dn_k}{dt} = A(1,5)6370 + A(2,4)38544 + A(3,3)44104.5 - A(1,6)1470 - A(2,6)1848$$

$$-A(3,6)6237 - A(4,6)9198 - A(5,6)1911 - A(6,6)441$$

At the same time, using Equation (9), the rate of change dn_k/dt of the number of banks n_k with asset size k is calculated per each year. Rewriting Equation (9) for the six-peer case, we have:

$$\frac{d_{nk}}{dt} = (-0.5){*}\frac{\sum_i^k ((\text{Ratio } of \log_{10}(\text{Number of banks})) - 1)}{\text{Total number of peer}}$$

$$i = \{1,\ 2,\ \ldots,\ 6\}, k = 6 \tag{A3}$$

In particular, using the data collected in Table II, i.e., for the year 2010, we have:

$$\frac{d_{nk}}{dt} = -0.003186014$$

At this point, the matrix equation (Equation (8)) can be constructed as follows:

$$\underbrace{\begin{bmatrix} \square & \cdots & \square \\ \vdots & \ddots & \vdots \\ \square & \cdots & \square \end{bmatrix}_{9\times 9}}_{\substack{A \\ n_i * n_k}} \times \underbrace{\begin{bmatrix} \square \\ \cdots \\ \square \end{bmatrix}_{9\times 1}}_{\substack{X \\ A(i,k)}} = \underbrace{\begin{bmatrix} \square \\ \cdots \\ \square \end{bmatrix}_{9\times 1}}_{\substack{B \\ \frac{dn_k}{dt}}}$$

Since the financial data refer to nine years, matrix A is a square matrix of dimension 9×9, which implies that both matrices X and B must have dimension 9×1. The unknowns of the matrix equation are the nine merging probability values that appear in each of the equations written for the years 2007-2015.

Matrices A and A^{-1}

These matrices indicate the number of banks with different asset sizes. Their entries are all known values that can be extracted from the current data, that is, using Table II for the year 2010 and the corresponding tables for the other years:

$$A = \begin{array}{c} 2007 \\ 2008 \\ 2009 \\ 2010 \\ 2011 \\ 2012 \\ 2013 \\ 2014 \\ 2015 \end{array} \begin{bmatrix} n_1n_5 & n_2n_4 & n_3n_3*0.5 & -n_1n_6 & -n_2n_6 & -n_3n_6 & -n_4n_6 & -n_5n_6 & -n_6n_6 \\ n_1n_5 & n_2n_4 & n_3n_3*0.5 & -n_1n_6 & -n_2n_6 & -n_3n_6 & -n_4n_6 & -n_5n_6 & -n_6n_6 \\ n_1n_5 & n_2n_4 & n_3n_3*0.5 & -n_1n_6 & -n_2n_6 & -n_3n_6 & -n_4n_6 & -n_5n_6 & -n_6n_6 \\ n_1n_5 & n_2n_4 & n_3n_3*0.5 & -n_1n_6 & -n_2n_6 & -n_3n_6 & -n_4n_6 & -n_5n_6 & -n_6n_6 \\ n_1n_5 & n_2n_4 & n_3n_3*0.5 & -n_1n_6 & -n_2n_6 & -n_3n_6 & -n_4n_6 & -n_5n_6 & -n_6n_6 \\ n_1n_5 & n_2n_4 & n_3n_3*0.5 & -n_1n_6 & -n_2n_6 & -n_3n_6 & -n_4n_6 & -n_5n_6 & -n_6n_6 \\ n_1n_5 & n_2n_4 & n_3n_3*0.5 & -n_1n_6 & -n_2n_6 & -n_3n_6 & -n_4n_6 & -n_5n_6 & -n_6n_6 \\ n_1n_5 & n_2n_4 & n_3n_3*0.5 & -n_1n_6 & -n_2n_6 & -n_3n_6 & -n_4n_6 & -n_5n_6 & -n_6n_6 \\ n_1n_5 & n_2n_4 & n_3n_3*0.5 & -n_1n_6 & -n_2n_6 & -n_3n_6 & -n_4n_6 & -n_5n_6 & -n_6n_6 \end{bmatrix}_{9\times 9}$$

To guarantee accuracy and quality of the numerical results, these data are transferred to an Excel spreadsheet. The results are represented in the following matrix:

$$A = \begin{bmatrix} 6{,}666 & 38{,}570 & 37{,}264.5 & -1{,}320 & -1{,}900 & -5{,}460 & -8{,}120 & -2{,}020 & -400 \\ 6{,}003 & 37{,}674 & 42{,}924.5 & -1{,}173 & -1{,}547 & -4{,}981 & -7{,}038 & -1{,}479 & -289 \\ 7{,}161 & 40{,}388 & 43{,}808 & -1{,}309 & -1{,}564 & -5{,}032 & -7{,}463 & -1{,}581 & -289 \\ 6{,}370 & 38{,}544 & 44{,}104.5 & -1{,}470 & -1{,}848 & -6{,}237 & -9{,}198 & -1{,}911 & -441 \\ 5{,}520 & 42{,}394 & 45{,}904.5 & -1{,}311 & -1{,}786 & -5{,}757 & -8{,}569 & -1{,}520 & -361 \\ 6{,}660 & 57{,}715 & 61{,}600.5 & -1{,}890 & -2{,}499 & -7{,}371 & -10{,}185 & -1{,}554 & -441 \\ 6{,}480 & 61{,}614 & 59{,}168 & -1{,}800 & -2{,}520 & -6{,}880 & -9{,}780 & -1{,}440 & -400 \\ 5{,}850 & 68{,}666 & 58{,}824.5 & -2{,}610 & -4{,}031 & -9{,}947 & -14{,}326 & -1{,}885 & -841 \\ 558 & 8{,}236 & 54{,}120.5 & -2{,}418 & -3{,}692 & -8{,}554 & -1{,}508 & -156 & -676 \end{bmatrix}$$

Hence, A^{-1} is calculated as follows:

$$A^{-1} = \begin{bmatrix} -0.000932619 & -0.002985411 & 0.002730619 & 0.00027741 & 0.003343458 & -0.004493852 & \\ -0.000423572 & -0.00010997 & 4.74567E-05 & 0.00080938 & -4.96889E-05 & -0.00138439 & \cdots \\ 0.000133259 & 0.002070442 & -0.000980614 & -0.000348641 & -0.00161355 & 0.001818217 & \\ -0.004805759 & -0.008210442 & 0.007729305 & 0.000823185 & 0.018609598 & -0.025199755 & \\ -0.01070761 & 0.000392614 & -5.50829E-05 & 0.020052954 & -0.005147657 & -0.026022772 & \cdots \\ 0.003201053 & 0.0287089 & -0.013601517 & -0.006140859 & -0.024313569 & 0.02867196 & \\ -0.002318961 & -0.00024026 & 0.000200531 & 0.004569195 & -0.000496806 & -0.007814561 & \\ -0.004449957 & -0.011553195 & 0.009634164 & 0.001257795 & 0.012931725 & -0.015943597 & \cdots \\ 0.046102508 & -0.170896775 & 0.066418919 & -0.063064225 & 0.140306993 & 0.015553422 & \end{bmatrix}$$

$$\begin{bmatrix} & & 0.00241669 & -9.29307E-05 & 0.000311574 \\ \cdots & \cdots & 0.001213711 & -6.99377E-05 & 4.78482E-05 \\ & & -0.001317159 & 0.000229205 & -0.000147565 \\ & & 0.013411997 & -0.000325168 & 0.001482281 \\ \cdots & \cdots & 0.023319864 & -0.00139087 & 0.000766669 \\ & & -0.020617252 & 0.003589279 & -0.002333069 \\ & & 0.006728234 & -0.000453076 & 0.000354087 \\ \cdots & \cdots & 0.008826862 & -0.000172058 & 0.001119268 \\ & & -0.02016419 & -0.018186993 & 0.006531766 \end{bmatrix}$$

Matrix *X*

This matrix consists of the merging probability values (i.e. $A(i, k-i)$, $A(i, k)$), which are the unknowns of the problem. The main objective of this study is to find the merging probability values in order to predict the next year behavior of the bank mergers:

$$X = \begin{bmatrix} A(1,\ 5) \\ A(2,\ 4) \\ A(3,\ 3) \\ A(1,\ 6) \\ A(2,\ 6) \\ A(3,\ 6) \\ A(4,\ 6) \\ A(5,\ 6) \\ A(6,\ 6) \end{bmatrix}$$

Matrix *B*

This matrix displays the rates of change (i.e. d_{nk}/dt) of number of banks with a given asset size in different one-year time intervals. These values are extracted from the data, that is, from

Table II for the year 2010 and the corresponding tables for the other years. They are computed using Equation (A3):

$$B = \begin{bmatrix} 0.007806956 \\ 0.005797105 \\ -0.004565698 \\ -0.003186014 \\ 0.00348063 \\ -0.01404075 \\ 0.001045032 \\ -0.010130251 \\ 0.078673686 \end{bmatrix}$$

Solving the multi-variable equation

The solution method implemented for the matrix equation is the one described by Equation (10). For the sake of completeness, we copy again the algebra steps below:

$$\begin{aligned} & A \times X = B \\ & A^{-1} \times A \times X = A^{-1} \times B \\ & I \times X = A^{-1} \times B \\ & X = A^{-1} \times B \end{aligned}$$

After some calculations, we obtain the solution matrix X below:

$$X = \begin{bmatrix} A(1,\ 5) \\ A(2,\ 4) \\ A(3,\ 3) \\ A(1,\ 6) \\ A(2,\ 6) \\ A(3,\ 6) \\ A(4,\ 6) \\ A(5,\ 6) \\ A(6,\ 6) \end{bmatrix} = \begin{bmatrix} 6.47753\text{E}{-05} \\ 1.82665\text{E}{-05} \\ -2.78223\text{E}{-05} \\ 0.000429496 \\ 0.000301283 \\ -0.000455575 \\ 0.000112501 \\ 0.000218185 \\ 0.000213908 \end{bmatrix}$$

where $A(i, j)$ is the probability that the banks in peer i merge with the banks in peer j.

Appendix 2. Data implementation for the reverse engineering method: years 2003-2011

$$A = \begin{bmatrix} 34{,}188 & 25{,}460 & 19{,}800.5 & -31{,}878 & -23{,}115 & -13{,}731 & -5{,}244 & -5{,}106 & -4{,}761 \\ 38{,}608 & 29{,}714 & 22{,}898 & -33{,}528 & -23{,}628 & -14{,}124 & -5{,}478 & -5{,}016 & -4{,}356 \\ 37{,}590 & 35{,}910 & 28{,}800 & -37{,}053 & -27{,}531 & -16{,}560 & -6{,}210 & -4{,}830 & -4{,}761 \\ 10{,}112 & 36{,}344 & 35{,}112.5 & -8{,}704 & -28{,}084 & -18{,}020 & -5{,}984 & -5{,}372 & -4{,}624 \\ 8{,}888 & 38{,}570 & 37{,}264.5 & -6{,}666 & -26{,}796 & -18{,}018 & -6{,}270 & -5{,}808 & -4{,}356 \\ 7{,}569 & 37{,}674 & 2{,}924.5 & -6{,}003 & -28{,}566 & -20{,}217 & -6{,}279 & -6{,}003 & -4{,}761 \\ 7{,}905 & 40{,}388 & 43{,}808 & -7{,}161 & -33{,}803 & -22{,}792 & -7{,}084 & -6{,}545 & -5{,}929 \\ 7{,}098 & 38{,}544 & 44{,}104.5 & -6{,}370 & -30{,}660 & -20{,}790 & -6{,}160 & -5{,}460 & -4{,}900 \\ 6{,}640 & 42{,}394 & 45{,}904.5 & -5{,}520 & -31{,}119 & -20, 907 & -6{,}486 & -5{,}727 & -4, 761 \end{bmatrix}$$

$$
A^{-1} = \begin{bmatrix}
-0.0001 & 0.0003 & -0.0001 & -0.0006 & 0.0004 & -0.0006 & 0.0002 & 0.0004 & -0.0000 \\
-0.0013 & 0.0014 & 0.0003 & -0.0001 & -0.0057 & 0.0025 & 0.0044 & -0.0113 & 0.0088 \\
0.0010 & -0.0008 & -0.0003 & -0.0012 & 0.0038 & -0.0022 & -0.0021 & 0.0065 & -0.0042 \\
-0.0001 & 0.0003 & -0.0002 & -0.0008 & 0.0008 & -0.0009 & 0.0002 & 0.0010 & -0.0003 \\
-0.0016 & 0.0017 & -0.0005 & -0.0016 & -0.0058 & 0.0023 & 0.0058 & -0.0128 & 0.0102 \\
0.0044 & -0.0034 & 0.0013 & -0.0035 & 0.0150 & -0.0089 & -0.0089 & 0.0260 & -0.0170 \\
-0.0089 & 0.0091 & 0.0020 & 0.0062 & -0.0446 & 0.0222 & 0.0270 & -0.0795 & 0.0595 \\
0.0004 & 0.0000 & -0.0002 & -0.0064 & 0.0074 & -0.0068 & 0.0006 & 0.0095 & -0.0045 \\
0.0013 & -0.0042 & 0.0003 & 0.0133 & 0.0092 & 0.0027 & -0.0177 & 0.0287 & -0.0295
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0.00960101 \\
0.009250039 \\
0.009850618 \\
-0.034417525 \\
-0.001193838 \\
-0.002736612 \\
0.006321434 \\
-0.008664714 \\
-0.000426524
\end{bmatrix}
$$

$$A \times X = B$$

$$A^{-1} \times A \times X = A^{-1} \times B$$

$$I \times X = A^{-1} \times B$$

$$X = A^{-1} \times B$$

The matrix solution

$$
\begin{bmatrix}
A(1,5) \\ A(2,4) \\ A(3,3) \\ A(1,6) \\ A(2,6) \\ A(3,6) \\ A(4,6) \\ A(5,6) \\ A(6,6)
\end{bmatrix}
= \begin{bmatrix}
-0.0001 & 0.0003 & -0.0001 & -0.0006 & 0.0004 & -0.0006 & 0.0002 & 0.0004 & -0.0000 \\
-0.0013 & 0.0014 & 0.0003 & -0.0001 & -0.0057 & 0.0025 & 0.0044 & -0.0113 & 0.0088 \\
0.0010 & -0.0008 & -0.0003 & -0.0012 & 0.0038 & -0.0022 & -0.0021 & 0.0065 & -0.0042 \\
-0.0001 & 0.0003 & -0.0002 & -0.0008 & 0.0008 & -0.0009 & 0.0002 & 0.0010 & -0.0003 \\
-0.0016 & 0.0017 & 0.0005 & -0.0016 & -0.0058 & 0.0023 & 0.0058 & -0.0128 & 0.0102 \\
0.0044 & -0.0034 & -0.0013 & -0.0035 & 0.0150 & -0.0089 & -0.0089 & 0.0260 & -0.0170 \\
-0.0089 & 0.0091 & 0.0020 & 0.0062 & -0.0446 & 0.0222 & 0.0270 & -0.0795 & 0.0595 \\
0.0004 & 0.0000 & -0.0002 & -0.0064 & 0.0074 & -0.0068 & 0.0006 & 0.0095 & -0.0045 \\
0.0013 & -0.0042 & 0.0003 & 0.0133 & 0.0092 & 0.0027 & -0.0177 & 0.0287 & -0.0295
\end{bmatrix}
* \begin{bmatrix}
0.00960101 \\
0.009250039 \\
0.009850618 \\
-0.034417525 \\
-0.001193838 \\
-0.002736612 \\
0.006321434 \\
-0.008664714 \\
0.000426524
\end{bmatrix}
$$

$$\begin{bmatrix} A(1,5) \\ A(2,4) \\ A(3,3) \\ A(1,6) \\ A(2,6) \\ A(3,6) \\ A(4,6) \\ A(5,6) \\ A(6,6) \end{bmatrix} = \begin{bmatrix} 2.31584E-05 \\ 0.000126649 \\ -2.55565E-05 \\ 2.29901E-05 \\ 0.000204959 \\ -0.000148732 \\ 0.000632991 \\ 0.00015493 \\ 0.000848175 \end{bmatrix}$$

Table AI. Aggregation probabilities: reverse engineering vs improved reverse engineering

Reverse engineering method		Improved reverse engineering method	
A(*i*,*k*–*i*), *A*(*i*,*k*)	Aggregation probabilities	*A*(*i*,*k*–*i*), *A*(*i*,*k*)	Aggregation probabilities
A(1, 5)	2.31584E–05	*A*(1, 4)	8.56526E–05
A(2, 4)	0.000126649	*A*(2, 3)	8.13121E–05
A(3, 3)	–2.55565E–05	*A*(1, 5)	0.00011107
A(1, 6)	2.29901E–05	*A*(2, 5)	0.000333539
A(2, 6)	0.000204959	*A*(3, 5)	0.000471192
A(3, 6)	–0.000148732	*A*(4, 5)	0.000121101
A(4, 6)	0.000632991	*A*(5, 5)	0.002087154
A(5, 6)	0.00015493	The rate of change is equal to (0.063412917≈0.063≈6.34%)	
A(6, 6)	0.000848175		

Table AII. Prediction and actual values for year 2012 using the improved approach

	Actual values		Prediction values	
Peer No.	No. of banks	Log_{10} (No. of banks)	No. of banks	Log_{10} (No. of banks)
1	85	1.93	89	1.95
2	93	1.97	90	1.96
3	110	2.04	110	2.04
4	360	2.56	355	2.55
5	522	2.72	515	2.71

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