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Time envelopment analysis: A new method for effectively incorporating time series in data envelopment analysis

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ABSTRACT

Data envelopment analysis (DEA) is a non-parametric tool for empirically evaluating the relative efficiency of homogeneous organizational units, i.e., decision-making units, by estimating the production frontiers. Time series analysis is a statistical technique that considers a series of data collected chronologically over time intervals. This study introduces a three-stage method, Time Envelopment Analysis (TEA), to effectively integrate time series analysis into DEA. The three-stage method includes a first-order autoregressive (AR(1)) model followed by DEA and ordinary least squares (OLS). The performance of the TEA method with four different values for the AR(1) parameters is compared with the DEA-OLS procedure using extensive Monte Carlo simulations. The simulation results show that the TEA method outperforms the DEA-OLS procedure. We further demonstrate that TEA is more accurate when the autoregressive parameter is smaller, particularly in scenarios defined by a progressive decrease in the impact of technical inefficiencies. We evaluate the proposed TEA method using a real-world healthcare dataset from 63 countries by estimating the effect of contextual variables on each country's productivity.

1. Introduction

Data envelopment analysis (DEA) is a popular performance comparison method developed by Charnes et al. (1978) and Banker et al. (1984) to evaluate efficiency in organizational units, i.e., decision-making units (DMUs). DEA measures the organizational performance of DMUs by identifying the inefficient ones and prescribing targets for improving their efficiencies without requiring any production function assumptions. DEA has attracted the interest of many researchers worldwide who are extending its theoretical foundations and applications in for-profit and non-profit organizations.

Given the widespread applicability of DEA, Banker and Natarajan (2008) proposed a two-stage DEA-ordinary least squares (OLS) procedure to examine the effect of contextual (explanatory) variables on productivity and provide statistically consistent estimators. The first stage applies DEA to measure the productivity score, while the second uses OLS to regress the logarithm of the score obtained in the first stage on contextual variables. The DEA-OLS procedure assumes independence between observations – the contextual variables could be correlated, but the authors required independence between contextual and input variables –. However, studies involving time series data involve serial correlations between observations, violating Banker and Natarajan's (2008) independence assumption. This source of friction adds up to the frontier modifications and efficiency correlations derived from the frontier estimation structure of DEA highlighted by Simar and Wilson (2011).

This study introduces a three-stage method, Time Envelopment Analysis (TEA), to effectively integrate time series analysis into DEA. In

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the first stage, a first-order autoregressive model (AR(1)) is used to generate time series values for a given set of noise and inefficiency variables. In other words, we introduce a temporal shock to the production process of DMUs whose effect either wears off or prevails over time, modifying their efficiency. The second stage employs DEA to measure the relative efficiency of DMUs. In the third stage, we apply OLS to the estimated productivity to assess the impact of the corresponding contextual variables.

Extensive Monte Carlo simulations are used to compare the performance of our proposed TEA method (i.e., the three-stage AR-DEA-OLS method) in the presence of time series data and the two-stage DEA-OLS method of Banker and Natarajan (2008). That is, we perform a rigorous statistical analysis to explore the impact of contextual variables on productivity by expressing output as a general function of the following two elements:

a. the inputs used, together with a series of contextual factors, and
b. an error term accounting for inefficiency and random noise.

The simulation results show that TEA outperforms the DEA-OLS procedure. We further demonstrate that TEA is more accurate when the autoregressive parameter is smaller.

Note that we are dealing with two primary sources of friction: the AR (1) process accounting for the effects of noise and technical inefficiency and the contextual variables. The first one may wear off or prevail as additional periods are considered. At the same time, the structure of the latter can be analyzed under the assumptions defined by Banker and Natarajan (2008), which, restrictive as they may be, fit the framework of the current analysis. The constraints imposed by Banker and Natarajan (2008) are thoroughly examined by Simar and Wilson (2011), who highlight their strong specificity. In the next section, we elaborate on the critique of the latter and the adequacy of the original constraints to the current analysis framework.

The specific combination of techniques composing TEA preserves the simplicity and applicability of the DEA-OLS framework defined by Banker and Natarajan (2008) while allowing for a direct analysis of temporal structural shocks to firms. Various dynamic extensions of DEA have been explored in the literature, mainly focusing on the forecasting capacities of time series (Emrouznejad et al., 2016). In this regard, TEA defines a tractable analytical framework highlighting the capacity of the model developed by Banker and Natarajan (2008) to evaluate structural efficiency when firms are subject to intertemporal shocks endowed with a specific stochastic structure. This feature constitutes a standard empirical phenomenon. The capacity of their model to assess the impact of contextual variables when the error term follows a structured autoregressive process, whose main components can also be estimated, constitutes one of the main results of the paper.

Following Banker and Natarajan (2008), we perform a series of simulations to generate the data used to evaluate the TEA-based estimator's performance employing a single-output, single-input cubic polynomial production function that is monotone and concave in the input range. We begin with one contextual variable and expand the analysis to incorporate two correlated variables when generating the output values through the simulation process. We assess the performance of TEA (with four different autoregressive parameters) and the DEA-OLS method to determine the impact of contextual variables on productivity. The mean absolute deviation (MAD) and root mean squared deviation (RMSD) are used for performance assessment.

1.1. Literature review

Several researchers have proposed models for using time series data in DEA. Chang and Mashruwala (2006) investigated whether the Bell system was a natural monopoly before its break-up using DEA and time series data. They treated each year as a distinct DMU and performed statistical tests to examine the returns to scale hypothesis. Silva et al. (2014) employed a DEA-based approach to determine which statistical measures are most appropriate for constructing an efficient fitness function. They used DEA to identify the best combination of statistical measures for building a more efficient fitness function for evolutionary algorithms. Lim (2016) used an inverse DEA model with time series analysis to create a novel prescriptive approach for establishing new product targets by considering anticipated changes on the production frontier.

Petridis et al. (2016) implemented a two-stage spatiotemporal DEA approach to evaluate how DMUs evolve. They assumed that DMUs consider the spatial and temporal dimensions to create the reference set for examining various points in time-related to the same DMU. These authors used DEA in the first stage and applied a multi-objective mixed integer linear programming model in the second stage. Petridis (2020) enhanced the DEA-time series literature by expanding Petridis et al.'s (2016) spatiotemporal DEA approach with undesirable inputs and outputs.

Salari and Khamooshi (2016) proposed an integrated fuzzy time series DEA approach for project performance prediction. They used fuzzy time series to estimate performance and DEA to evaluate efficiency. The mean square error, mean absolute error, and mean absolute percentage error measures were used to select the superior projects. We have not found a reference in the literature that incorporates time series in DEA to investigate the impact of contextual variables on productivity, confirming the originality of the TEA method proposed in this study.

Banker and Natarajan (2008) included both exogenous and organizational variables – under the control of the DMUs – as part of the contextual ones. Exogenous variables condition both efficiency differentials across DMUs and the subsequent managerial policies. Several variants of the two-stage method based on modifications of the techniques implemented at each stage have been introduced in the literature to study the effect of contextual variables on efficiency.

Bădin et al. (2012) applied conditional efficiency measures in the first stage to assess the effect of environmental factors on the production process. Daraio and Simar (2014) extended the model of Bădin et al. (2012) into a robust directional distance setting and evaluated the importance of environmental factors on the resulting efficiencies via bootstrapping. Furthermore, Esteve et al. (2020) defined a technique based on regression trees combined with cross-validation to estimate production frontiers.

Among the novel techniques implemented in the second stage, Souza and Gomes (2015) applied a fractional nonlinear regression model with a dynamic Generalized Method of Moments estimation. Recently, Rebai et al. (2020) followed a machine-learning approach and used regression trees and random forests to quantify the impact of contextual variables.

Finally, Bădin et al. (2014) reviewed the literature dealing with environmental variables in non-parametric frontier models while focusing on their analysis of the impact of conditional efficiency scores. Parmeter and Zelenyuk (2019) reviewed the literature on semi- and non-parametric estimators for stochastic frontier models that explicitly allow for statistical noise and their interactions with traditional DEA models.

1.2. Contribution

The stochastic framework proposed in this paper considers a scenario where the structural productive capacity of DMUs is subject to either shocks of decreasing intensity or a noisy AR(1) process that prevails through the analysis period. In the former case, shocks lack a persistent stochastic structure and consist of decreasing temporal frictions that vanish after a series of periods, distorting the effect of the contextual variables on the production process of DMUs. Note that if the frictions were permanent and constant, the performance of the resulting TEA model would coincide with that of DEA-OLS, a quality that is illustrated numerically. In this regard, the performance of TEA improves when endowing the errors with a stochastic white-noise structure, providing the DEA estimator with an identifiable distribution of the errors, a feature described in Proposition 6 in Banker (1993).

The dynamic behavior of DMUs when subject to persistent structural disruptions constitutes a phenomenon that remains mainly unaddressed in the DEA literature despite the substantial variety of potential applications, which range from uncertain scenarios (Mahmoodirad et al., 2023) to healthcare (Azadi et al., 2023) and environmental (Sun et al., 2024) settings. In particular, the structural qualities determining the production process of a firm are subject to inherent frictions and shocks that condition its efficiency and whose relative importance and persistence should be carefully analyzed, particularly when facing capacity constraints (Orisaremi et al., 2025) within supply chain environments (Lin and Lu, 2024) and sequential scenarios subject to strategic interactions (Zhang et al., 2024). Furthermore, a variety of applications in complementary evaluation environments arise when considering interactions with consumers (Liu et al., 2022), their ability to assimilate disruptions to the supply of services (Spotts et al., 2022), or account for intangible output features such as corporate social responsibility (Yao and Zhao, 2024).

We discuss below the critique of Simar and Wilson (2007; 2011), who emphasized the limited applicability of the framework proposed by Banker and Natarajan (2008) and proposed a series of alternative techniques to soften the restrictions imposed by Banker and Natarajan (2008) in their model. On the other hand, the capacity of the method introduced by Banker and Natarajan (2008) to identify the structural effect of contextual variables using standard econometric techniques enhances its applicability. The academic discussion between supporters and detractors of both approaches has prevailed over the years (Camanho et al., 2024; Moradi-Motlagh and Emrouznejad, 2022; Emrouznejad and Yang, 2018).

We acknowledge the limitations of the model of Banker and Natarajan (2008), as highlighted by Simar and Wilson (2011), regarding statistical inference. However, we will focus on the capacity of DEA to identify the structural and contextual performance of DMUs when subject to decreasing shocks of limited duration or stochastic patterns where noise prevails throughout the analysis. Simar and Wilson (2011) highlighted several qualities limiting the general applicability of the method presented by Banker and Natarajan (2008). However, as illustrated below, standard manufacturing environments describing the performance of DMUs through time fit easily within the analysis framework in the same way they do within DEA environments when evaluating efficiency. A summary of the main restrictions that Simar and Wilson (2011) listed follows.

a. First, contextual variables have a negative structural effect on the productive capacity of DMUs coupled with temporal shocks – both positive and negative – to the production process that condition its efficiency. The former quality is structural, while the latter is temporal and may prevail or decrease over time.
b. Second, the independence between the contextual and output variables implies that the production function is subject to structural frictions – such as depreciation – that affect the productive capacity of DMUs independently of their input consumption. Banker and Morey (1986) allow for dependence between the contextual and input variables, with the former affecting both efficiency and the frontier. We will simulate this latter scenario numerically to illustrate the capacity of TEA to identify contextual effects when, for instance, depreciation rates condition the usage of inputs and the productive capacity of DMUs.
c. Third, the homoskedasticity and homogeneity of the random noise v_t and technical inefficiency u_t, $t = 1, \cdots, T$, variables are satisfied by considering the dynamic performance of individual DMUs per DEA implementation. That is, firms are evaluated independently so that their inherent characteristics determine the effect of contextual variables while being subject to exogenous dynamic efficiency shocks. DEA is, therefore, not applied to a variety of firms whose efficiencies are then compared and used to assess contextual effects but to individual firms evaluated at different points in time. In other words, the current framework does not apply to panels of longitudinal data describing the evolution of several DMUs across multiple periods.
d. Fourth, the model considers temporal shocks to the productive capacity of DMUs whose effect either prevails as an AR(1) process or vanishes through time. The stochastic structure of the model fits within the symmetrically bounded noise framework defined by Gstach (1998) and required by Banker and Natarajan (2008). We will analyze the capacity of TEA to identify the effect of contextual variables when considering a variety of frictions of different persistence.
e. Fifth, the data-generating process implies that the structural quality inherent to the contextual variables modifies the production frontier. At the same time, the production process is also subject to stochastic shocks of a temporal nature that affect the efficiency of DMUs. Thus, while, as noted by Simar and Wilson (2011), structural, contextual constraints impose a monotonic decrease in the productive capacity [frontier] of firms for any input level – violating their separability condition –, it is the exogenous temporal shocks the ones having a persistent or vanishing effect on their efficiency.

Banker and Natarajan (2008) assumed that the random noise and technical inefficiencies are not under the control of the DMU. However, the empirical evidence illustrates how DMUs can smooth some of these effects through time, particularly those arising from technical inefficiencies (Alsaleh et al., 2017). As we will demonstrate numerically, the production structure would collapse if these effects could not be controlled and smoothed. The capacity of DMUs to manage part of these frictions will also provide intuition regarding implementing other random processes such as Moving Average (MA) or Autoregressive Moving Average (ARMA) models.

We conclude by noting that a series of Monte Carlo simulations have been performed to illustrate the results numerically while acknowledging the importance of bootstrapping—highlighted by Simar and Wilson (2007; 2011)—to evaluate the effect of [separable] contextual variables on firms' production processes.

The remainder of the paper is structured as follows. Section 2 presents the three-stage TEA method. Section 3 describes the Monte Carlo simulation method used to compare the performance of the TEA and DEA-OLS methods. Section 4 presents a case study to demonstrate the applicability of TEA, and Section 5 concludes and suggests future research directions.

2. TEA method

The TEA method proposed in this study is composed of three stages shown in Fig. 1:

2.1. Stage 1: First-order autoregressive method

A time series model that forecasts the value of successive periods is regressed using the observations from previous periods as input. The number of observations used for prediction determines the order of the model. The notation $AR(p)$ designates an autoregressive model of order p:

$$AR(p) : A_t = \phi_0 + \sum_{i=1}^{p} \phi_i A_{t-i} + \varepsilon_i$$

where $\phi_1, \cdots, \phi_p$ are the autoregressive parameters, ϕ_o is a constant, and ε_i is white noise. The first-order autoregressive model, AR(1), uses only the previous period to predict the current one. This study uses the AR(1) model for generating data.

The empirical literature has extensively documented the improvements in the technical efficiency of DMUs triggered by different market forces within a variety of industries, such as, for instance, increased

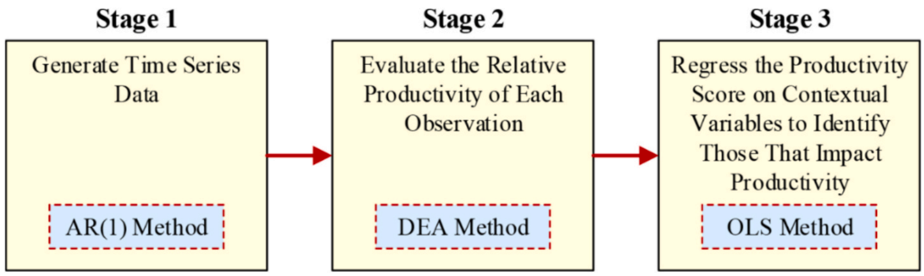


Fig. 1. Proposed TEA method.

competition in the Norwegian banking industry (Berg et al., 1991), increased managed care insurance in US hospitals (Shelton Brown, 2003), mergers and acquisitions in the Japanese regional banking sector (Halkos et al., 2016), adoption of new strategies and modern technology in the bioenergy industry within the European Union (Alsaleh et al., 2017), access to credit for rice farmers in Bangladesh (Jimi et al., 2019), ownership type in several Chinese industrial sectors (Walheer and He, 2020), coping and adaptation strategies of farmers in Ghana (Ankrah Twumasi and Jiang, 2021), and the quality of Information and Communication Technologies in a panel sample covering 100 countries (Ndubuisi et al., 2022).

Alsaleh et al. (2017) illustrated how the European Union smooths technical inefficiencies through time, contrasting with the more pervasive effects of organizational or structural shocks. Similarly, Sandvold (2016) described a cumulative dynamic impact of learning by doing on technical efficiency. Both exogenous and endogenous factors interact within the contextual variables, allowing DMUs to improve their technical efficiency while facing organizational and managerial frictions and generating noise.

The production structure of the DMUs is conditioned by the magnitude and persistence of the different shocks and the subsequent noise. Given the empirical evidence presented in the literature, we will assume that DMUs can smooth technical inefficiency frictions or that positive technical efficiency shocks are short-lived. These features provide intuition regarding including an AR(1) process to describe the persistence of technical inefficiency shocks. This is not the case for contextual variables, reflecting the effect of exogenous and managerial frictions that prevail through time. In the paper of Banker and Natarajan (2008), DMUs cannot control the behavior of the shocks derived from contextual variables or technical inefficiency.

Note that MA or ARMA models would impose cumulative – technical inefficiency – frictions similar to those derived from an increment in the order of the AR process. For instance, consider a standard $MA(q)$ process.

$$MA(q) : X_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

where μ defines the mean of the series for a given set of parameters, θ_i, $i = 1, \cdots, q$, and the white noise error terms, $\varepsilon_t, \cdots, \varepsilon_{t-q}$. We will illustrate through the numerical simulations that introducing a higher-order AR process into the production framework highlights the negative consequences of the accumulation of friction. Indeed, this feature would consistently lead to zero outputs. The intuition is simple: if DMUs cannot contain technical inefficiencies through the production process, the persistent nature of these cumulative frictions would collapse their production structure and the subsequent identification capacity of the model.

The same type of intuition applies to a standard ARMA model, such as:

$$ARMA(p, q) : X_t = \varepsilon_t + \sum_{i=1}^{p} \phi_i X_{t-i} + \sum_{i=1}^{q} \omega_i \varepsilon_{t-i}$$

Clearly, the shocks in Banker and Natarajan's (2008) model do not accumulate due to its static nature.

2.2. Stage 2: DEA method for productivity measurement

Consider time series observations in $t = 1, \cdots, T$ periods, where:

- $\boldsymbol{Y}_t \equiv (y_{1t}, \cdots, y_{Rt})$ is an output vector in the t^{th} period,
- $\boldsymbol{X}_t \equiv (x_{1t}, \cdots, x_{It})$ is an input vector in the t^{th} period, and
- $\boldsymbol{Z}_t \equiv (z_{1t}, \cdots, z_{St})$ is a vector of contextual variables in the t^{th} period.

Farrell's (1957) considered this setting by assuming:

- $\boldsymbol{Y}$ as a farm's output in tons of grain,
- $\boldsymbol{X}$ As a farm's input for labor, capital, and materials, and
- $\boldsymbol{Z}$ as a farm's contextual variables for ownership and management.

We present a single output (y_t) model by assuming that the time series data in the t^{th} period is generated using the production function $\varphi(\boldsymbol{X}_t)$ with an ε_t error term. The production function $\varphi(\boldsymbol{X}_t)$ is characterized as monotone increasing and concave in $\boldsymbol{X}_t$ for all $t = 1, \cdots, T$. Vector $\boldsymbol{X}_t$ is related to a single output y_t through the production function as follows:

$$y_t = \varphi(\boldsymbol{X}_t) * e^{\varepsilon_t^*} \tag{1}$$

where error ε_t^* generated in the process is specified as follows:

$$\varepsilon_t^* = v_t - u_t - \sum_{s=1}^{S} \beta_{st} z_{st} \tag{2}$$

Here, v_t is random noise bounded above with a two-tailed distribution, u_t is technical inefficiency with a one-tailed distribution and z_{st} represents all positive contextual variables for $t = 1, \cdots, T$. The measurement of contextual variables is performed using the weights $\beta_{st}, s = 1, \cdots, S$, and $t = 1, \cdots, T$, which are all positive, implying that the inefficiency of the observation increases as the value of the contextual variables increases.

The error term in the t^{th} period, $\varepsilon_t = v_t - u_t$, is the sole result of noise and technical inefficiency. The probability density functions associated with the different variables are assumed to be given by the expressions (3a) to (3d):

$$f_{x_{it}}(x_{it}) = 0 \forall x_{it} < 0, \forall t \tag{3a}$$

$$f_{z_{st}}(z_{st}) = 0 \forall z_{st} < 0, \forall t \tag{3b}$$

$$f_{u_t}(u_t) = 0 \forall u_t < 0, \forall t \tag{3c}$$

$$f_{v_t}(v_t) = 0 \forall |v_t| > V^M, \forall t \tag{3d}$$

We are imposing an upper bound V^M on the noise, which will determine the domain of reference when defining the joint distribution of the composed error ε_t. Note that these conditions prevent the emergence of negative inputs, contextual variables, and technical inefficiencies. Therefore, the last two variables will have a direct negative effect on the output produced.

We assume that vectors $\boldsymbol{X}_t$ and $\boldsymbol{Z}_t$, the noise v_t, and the inefficiency u_t, are independently distributed. We impose no restrictions on the components of the input vector $\boldsymbol{X}_t$, which may be correlated with each other or the contextual variables vector $\boldsymbol{Z}_t$, which may also be correlated with each other. The variance of each stochastic variable is finite, and $E(v_t) = 0, \forall t$. As confirmed by inspection, the probability density function of the error in the t^{th} period, ε_t, is given by:

$$f_{\varepsilon_t}(\varepsilon_t) = \int_{\varepsilon_t}^{V^M} f_{v_t}(v_t) f_{u_t}(v_t - \varepsilon_t) dv_t = \int_{0}^{V^M - \varepsilon_t} f_{u_t}(u_t) f_{v_t}(u_t + \varepsilon_t) du_t \tag{4}$$

As was the case in Banker and Natarajan (2008), we applied the stochastic structure defined by Gstach (1998) to the error term and imposed an upper bound on the noise.

2.2.1. Estimating the impact of contextual variables

The relationship in Eq. (1) can be transformed into Eq. (5) by specifying $\varphi(\boldsymbol{X}_t)$ as $\varphi(\boldsymbol{X}_t;\gamma)$, where γ is a parameter vector:

$$\ln y_t = \ln\varphi(\boldsymbol{X}_t;\gamma_t) - \sum_{s=1}^{S} \beta_{st} z_{st} + \varepsilon_t \tag{5}$$

If $\varphi(\bullet)$ is the Cobb-Douglas function, the following holds:

$$\ln y_t = \gamma_{0t} + \sum_{i=1}^{l} \ln\gamma_i x_{it} - \sum_{i=1}^{S} \beta_{it} z_{it} + \varepsilon_t \tag{6a}$$

while if $\varphi(\bullet)$ is the translog function, the following holds:

$$\ln y_t = \gamma_{0t} + \sum_{i=1}^{l} \gamma_i \ln x_{it} + \sum_{i=1}^{l} \gamma_{ii} (\ln x_{it})^2 + \sum_{\substack{i,k=1 \\ i<k}}^{l} \gamma_{ik} \ln x_{it} \ln x_{kt} - \sum_{i=1}^{S} \beta_i z_{it} + \varepsilon_t \tag{6b}$$

According to Schmidt (1976), applying the OLS estimation procedure is justified when the parametric form is known since ε_t has a finite variance and even though its mean may differ from zero. Moreover, since the OLS estimators of β_i are consistent, we may use the t-statistic to determine if a certain contextual variable impacts productivity.

Pitt and Lee (1981) and Kalirajan (1989) developed a two-stage parametric procedure in which the first stage estimates $\ln y = \ln\varphi(X;\gamma) + \varepsilon^*$, while the second stage estimates $\widehat{\varepsilon}^* = -\sum_{i=1}^{S}\beta_i z_i + \varepsilon$, where $\widehat{\varepsilon}^*$ represents the residual estimated in the first stage. For the parametric case, Wang and Schmidt (2002) illustrated that a parametric procedure in one stage is more suitable for evaluating inefficiency and the impact of contextual variables on productivity.

2.2.2. Specifying monotone increasing and concave production functions

The TEA method proposed applies an AR(1) model in the first stage to regress the time series values on the previous values from the same series. The second stage uses DEA to assess the relative efficiency of DMUs. In the third stage, an extensive simulation demonstrates how the proposed TEA (namely, the three-stage AR-DEA-OLS) method outperforms the two-stage DEA-OLS method.

Assume the time series data are obtained through an AR(1) process. The second stage involves applying the following DEA-based estimation procedure to the available data (see Gstach, 1998):

$$\ln\widetilde{\varphi}(\boldsymbol{X}_t) = \ln\varphi(\cdot) + V^M \tag{7a}$$

$$\ln\widetilde{\theta} = (\varepsilon - V^M) - \sum_{i=1}^{S} \beta_{it} z_{it} = (v - V^M) - u - \sum_{i=1}^{S} \beta_{it} z_{it} \leq 0 \tag{7b}$$

Substituting Eqs. (7a) and (7b) into Eq. (1) leads to the following equation:

$$\ln y_t = \ln\widetilde{\varphi}(\boldsymbol{X}_t) + \ln\widetilde{\theta} \tag{8}$$

Since $\widetilde{\theta} \leq 1$ the last equation is similar to the standard DEA model (Banker 1993). Note also that the productivity from the second stage can be defined in terms of the contextual variables by comparing Eqs. (1), (7b), and (8):

$$\ln\widetilde{\theta} = -\sum_{i=1}^{S} \beta_{it} z_{it} - \widetilde{\varepsilon} \tag{9}$$

where $\widetilde{\varepsilon} = V^M - \varepsilon \geq 0$. A comparison of Eq. (9) with the conventional parametric definition of production frontier highlights the similarity between both specifications (Aigner and Chu, 1968). Therefore, given that OLS methods are appropriate for parametric production frontier estimation, they should deliver consistent estimators of the β parameters in Eq. (9).

2.3. Stage 3: OLS method

Let us define $\beta_0 = E(\varepsilon) - V^M$ and $\delta = \varepsilon - E(\varepsilon)$, and rewrite Eq. (9) as:

$$\ln\widetilde{\theta} = \beta_0 - \sum_{i=1}^{S} \beta_{it} z_{it} + \delta \tag{10}$$

We then replace $\ln\widetilde{\theta}$ with $\ln\widehat{\widetilde{\theta}}$(namely, the DEA estimator of true productivity) in Eq. (10). Consistency in the evaluation of the impact of the contextual variables is preserved by the use of $\ln\widehat{\widetilde{\theta}}$ rather than the true $\ln\widetilde{\theta}$.

Proposition 1. *(If a positive definite matrix is defined as* $\mathbf{Q} = \text{plim}(Z'Z/n)$*, then, the OLS estimator of* $\widetilde{\beta}$ *in Eq. (11) produces an estimator of the parameter vector* β *that is consistent (Banker and Natarajan, 2008, p. 51):)*

$$\ln\widehat{\widetilde{\theta}} = \widetilde{\beta}_0 - Z\widetilde{\beta} + \widetilde{\delta} \tag{11}$$

Therefore, the use of OLS after DEA to assess the effect of the contextual variables is valid under our assumptions.

Next, we consider Propositions 5 and 6 in Banker (1993) for the third stage estimation of TEA. Proposition 5 shows that the DEA estimator is consistent, while Proposition 6 demonstrates that the estimator retrieves the true distribution of the error term shifted upward (Schmidt, 1976). Assuming that the error is distributed as an AR(1) process, the standard methods for estimating AR(1) processes should work well in this context. These methods take the first or weighted difference using the estimated serial correlation coefficient. Thus, we calculate the serial correlation (r) of the estimated residual terms and then define τ – which will be equal to the estimator of the error $\widehat{\varepsilon}$ –:

$$\tau_t = \varepsilon_t - r(\varepsilon_{t-1}) \tag{12}$$

In AR(1) processes, the variable is presented as a linear function of its value in the previous period, and the error term (white noise) e_t:

$$\varepsilon_t = \phi_0 + \phi\varepsilon_{t-1} + e_t \tag{13}$$

The white noise follows a normal distribution with zero mean and constant variance. When the white noise value is different from zero, the variable is influenced by the value of the white noise infinitely far into long-standing time. When the value of the parameter (ϕ) is equal to zero, the variable is equivalent to e_t; when ϕ is equal to one and the intercept (ϕ_0) is equal to zero, and the variable is equal to a random walk. Although the relationship between the value of the variable in periods t and $t-1$ can be measured using the autocorrelation function, this study considers zero intercepts together with $\phi \in \{0.25, 0.50, 0.75, 1\}$ values.

Note that we are introducing a series of correlated errors into the regression model, defining the third stage. This correlation may bias the quality of the regression in terms of explainability but does not affect its capacity to estimate the parameter vector β. The standard Prais–Winsten estimation could be applied to account for the error correlation after identifying the type of AR process generating the data. In a related environment within a panel data framework, Souza and Gomes (2015) dealt with the correlations among the efficiency measurements of DMUs caused by the scores being defined relative to the efficient DMUs on the DEA/Free Disposal Hull (FDH) frontier. Extensions of the current model could be defined to incorporate both approaches into the analysis when dealing explicitly with correlation effects.

3. Simulation study

In this section, we run a series of Monte Carlo simulations to assess the performance of the procedures in two and three stages regarding the impact of the contextual variables on productivity. The intuition provided in the previous section implies that we should expect TEA to outperform DEA-OLS when time series data are present. The simulation results will help us evaluate the performance of the TEA method relative to DEA-OLS.

3.1. Design of the simulation study

We duplicate Banker and Natarajan's (2008) experiment design to facilitate comparisons between methods. We use simulations to assess the three-stage TEA procedure's performance and investigate contextual variables' impact on productivity.

The data for our simulations are generated using the same production function $\varphi(x_t)$ as Banker and Natarajan (2008):

$$\varphi(x_t) = -37 + 48x_t - 12x_t^2 + x_t^3 \tag{14}$$

We use a uniform distribution in the interval $[1, 4]$ to generate the input variable x_t. Fig. 2 illustrates the production function, which is continuous, monotone increasing, and concave within the chosen interval [1,4]. Other continuous, monotone-increasing, and concave production functions would deliver almost identical results.

In the initial simulation, we consider one contextual variable z_t for the t^{th} period generated using a uniform distribution in the [0,1] interval. The coefficient β_t that describes the impact of z_t on productivity is set to 0.2. The noise variable ν_t is generated using a normal distribution with zero mean that is two-side truncated, where $6\sigma_{\nu_t}$ and $-6\sigma_{\nu_t}$ define the upper and lower bounds, respectively. We present the initial results for $\sigma_{\nu_t} = 0.04$. A half-normal distribution with $\sigma_{u_t} = 0.15$ is used to generate the inefficiency variable u_t. We then draw the random input, contextual, noise, and inefficiency variables from the above probability distributions – which are assumed to be independent – and compute the logarithm of the output y_t as $\ln y_t = \ln(-37 + 48x_t - 12x_t^2 + x_t^3) - 0.2z_t + \nu_t - u_t$. We repeat the procedure to generate output for each new variable draw.

We have chosen the values of the parameters specifying the composed error distribution according to the values defined in previous studies that used data generation processes dealing with composed error distributions (Olson et al., 1980; Banker et al., 2004, 1993; and Banker and Natarajan, 2008). In our simulation, the ratio between the variance of u and ν equals 5.23, corresponding to moderate measurement error scenarios in previous studies. The expected value of efficiency $E(e^{-u})$ equals 0.83 with 71.6 % (62.4 % in Banker and Natarajan, 2008) of the variance of $\ln\left(\frac{y}{\varphi(x)}\right)$ following from the inefficiency term, 21.3 % (25.4 % in Banker and Natarajan, 2008) from the contextual variable, and the remaining 7.1 % from the measurement error.

Finally, we note that an FDH model imposes fewer restrictions than DEA on the true production frontier; namely, it does not require the production function to be concave. In this regard, Esteve et al. (2020) built a classification method starting from an FDH framework – despite the reticence expressed by De Borger and Kerstens (1996) regarding the use of radial measures in FDH environments –. Given its flexibility, we also implement an FDH framework to measure productivity in Stage 2 and compare the results obtained with those of the DEA-based model in both the concave and non-concave cases.

To do so, we modify the shape of the production function used in the simulations. The function considered by Banker and Natarajan (2008) is defined for values of the input variable distributed in [1,4], leading to a concave production framework that delivers outputs in [0,27]. We consider a convex production function defined within the same numerical setting for comparison purposes.

$$\varphi(x_t) = \frac{-x_t + x_t^3}{2.2222} \tag{15}$$

Note that when the input variable is defined in [1,4] the convex production function delivers values within the interval [0, 27.00027]. The upper limit of the interval converges to a value of 27 when further decimals are added to the denominator. However, the continuity of the uniform density from which the inputs are drawn implies that a realization of 4 takes place with zero probability.

We have considered this function as being fully aware of the resulting consistency problems. At the same time, it allows us to study a set of efficiency scenarios that remain outside the scope of traditional DEA models. As intuition suggests, the accuracy of the results will weaken. Moreover, the suboptimal performance of the FDH setting, particularly in the convex case, will be used to validate the

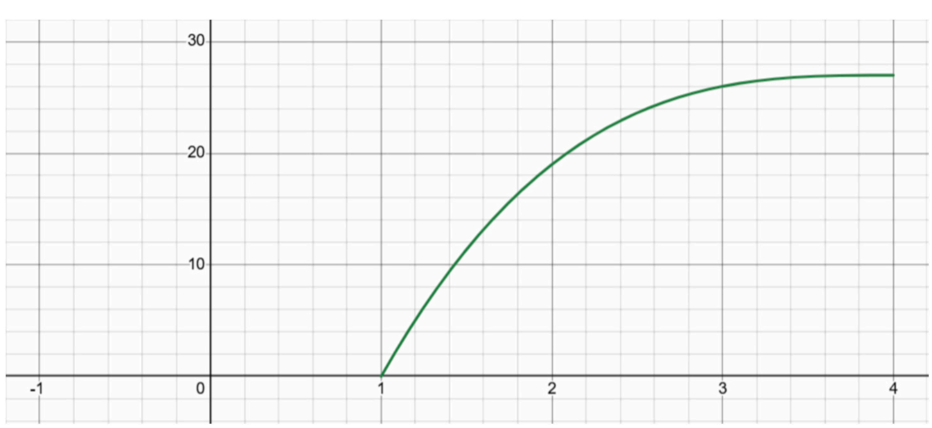


Fig. 2. The assumed production function..$\varphi(x_t) = -37 + 48x_t - 12x_t^2 + x_t^3$

implementation of the DEA-OLS and TEA models.

3.2. Data generation

We generate an initial experiment (x_t, y_t) for $t = 1, \cdots, N$ absent the AR(1) model. Then, we use the process $\varepsilon_t = \phi\varepsilon_{t-1} + e_t$, $\phi \in \{0.25, 0.50, 0.75, 1\}$, to generate four more experiments and denote the corresponding output by y_t^{ϕ}. That is, the extended data generation processes include four experiments $\left(x_t, y_t^{\phi}\right), \forall\phi \in \{0.25, 0.50, 0.75, 1\}$, conducted according to the assumptions described above. We run the Banker-Charnes-Cooper (BCC) model for each experiment, including single input and single output data.

We analyze the results obtained by simulating two different AR(1) models. The first one includes a white noise error term distributed as $e_t \sim N(0, \sigma_e = 0.04)$. We have based the standard deviation of the white noise process on the coefficient assigned to the contextual variable by Banker and Natarajan (2008); that is, noise realizations are bounded within $[-0.2, 0.2]$ when simulating $\varepsilon_t = \phi\varepsilon_{t-1} + e_t$. The second AR(1) model omits the white noise term and considers deterministic decreasing errors given by $\varepsilon_t = \phi\varepsilon_{t-1}$. We will illustrate how, despite the addition of noise and the subsequent increase in the variance of the AR (1) process, the stochastic patterns resulting from e_t improve the values of the mean absolute deviation (MAD) and root mean squared deviation (RMSD) percentages relative to the scenario with vanishing inefficiency shocks.

We have also simulated AR models of different order for the error term variable. Intuitively, the length of the AR process can be interpreted as the persistence of the stochastic shocks – triggered by technical inefficiencies and frictions – suffered by the DMUs through their production process. For instance, consider the following AR(2) model:

$$\varepsilon_t = \phi_1\varepsilon_{t-1} + \phi_2\varepsilon_{t-2} + e_t \tag{16}$$

We have defined combinations of values from the set $\phi \in \{0.25, 0.50, 0.75, 1\}$ while assuming that $\phi_2 \le \phi_1$, that is, the effect of the shock vanishes through time.

An immediate effect of this extension is a rapid increment of the error term that leads to zero outputs, preventing the computation of the efficiency scores. This is the case even when assigning a value of 0.25 to both AR(2) coefficients. Thus, for the model to work, we should consider much smaller values for both AR coefficients, namely, a relatively low persistence of the shocks, leading to a similar intuition as the one derived from the assumption of an AR(1) model. Clearly, the same problem arises when implementing AR models of higher order. From an intuitive viewpoint, decreasing the ability of firms to contain the effects of stochastic noise and inefficiency shocks weakens the capacity of the model to evaluate their behavior.

3.3. Estimation methods

We apply OLS to examine the impact of the contextual variable z on productivity using different batches of simulated data. The results obtained allow us to compare our three-stage TEA with the DEA-OLS procedure proposed by Banker and Natarajan (2008). The AR(1) stage delivers time series data generated via different autoregressive parameters, i.e., $\phi \in \{0.25, 0.50, 0.75, 1\}$. In the second stage, we apply the BCC linear program (Banker et al., 1984) to calculate the efficiency, $\widehat{\widehat{\theta}} \le 1$, of each sample of input–output data (x_t, y_t) and $\left(x_t, y_t^{\phi}\right)$, $\forall\phi \in \{0.25, 0.50, 0.75, 1\}$, $t = 1, \ldots, N$. In particular, the DEA model applied in the simulation adapts the standard BCC with variable returns to scale to the current dynamic scenario. The productivity score of the jth DMU in a sample of N observations within each of the 2,000 iterations described below is given by

$$\widehat{\widehat{\theta}}_j = \operatorname{argmax}\left\{\widehat{\widehat{\theta}} \mid \sum_{t=1}^{N}\lambda_t y_t \ge \widehat{\widehat{\theta}} y_j; \sum_{t=1}^{N}\lambda_t x_t \le x_j; \sum_{t=1}^{N}\lambda_t = 1; \lambda_t \ge 0 \forall t = 1, \cdots, N\right\} \tag{17}$$

The OLS stage estimates the relationship $\ln\widehat{\widehat{\theta}} = \beta_0 - \beta z + e_0$.

3.4. Performance evaluation

Our experiment consists of 400,000 observations (2,000 sets × 200 observations in each set). We run 2,000 iterations to estimate the impact of contextual variables on productivity by applying the above-described methods to a data set of 200 observations per iteration. Thus, the number of β values estimated per method equals 2,000. We compare the performance of each method through two error metrics: MAD and RMSD percentages. MAD is defined as follows:

$$100 \times \frac{1}{0.2}\left(\frac{1}{2000}\sum_{t=1}^{2000}\left|\widehat{\beta}_t - 0.2\right|\right) \tag{18}$$

while RMSD is given by:

$$100 \times \frac{1}{0.2}\left(\frac{1}{2000}\sum_{t=1}^{2000}\left(\widehat{\beta}_t - 0.2\right)^2\right)^{1/2} \tag{19}$$

where $\widehat{\beta}_t$ is the estimated value of β_t for iteration t.

3.5. Simulation results

The results obtained in the TEA ($\varepsilon_t = \phi\varepsilon_{t-1} + e_t$) setting are described in Table 1. Substantially inferior outcomes are obtained within an FDH scenario, particularly in the convex case, where TEA performs worse than DEA-OLS. These examples illustrate the suboptimal performance of the model when the concavity assumption is relaxed and a convex production function is assumed. The outcomes obtained from TEA and DEA-OLS are similar when considering a concave FDH scenario but markedly inferior to those derived from a standard BCC framework.

Fig. 3 provides a graphical representation of the results from Table 1, offering visual insight and enhancing the intuitive understanding of the comparison.

As illustrated in Table 2, decreasing the sample size from 200 to 50 simulations worsens all results considerably, particularly those obtained in the FDH scenario.

The performance results of TEA absent white noise ($\varepsilon_t = \phi\varepsilon_{t-1}$) and DEA-OLS are summarized in Table 3. As in the case of white noise, the results show that TEA with $\phi = 0.25$ delivers the best performance. Note also that the outcome of TEA with $\phi = 1$ is almost identical to that of DEA-OLS.

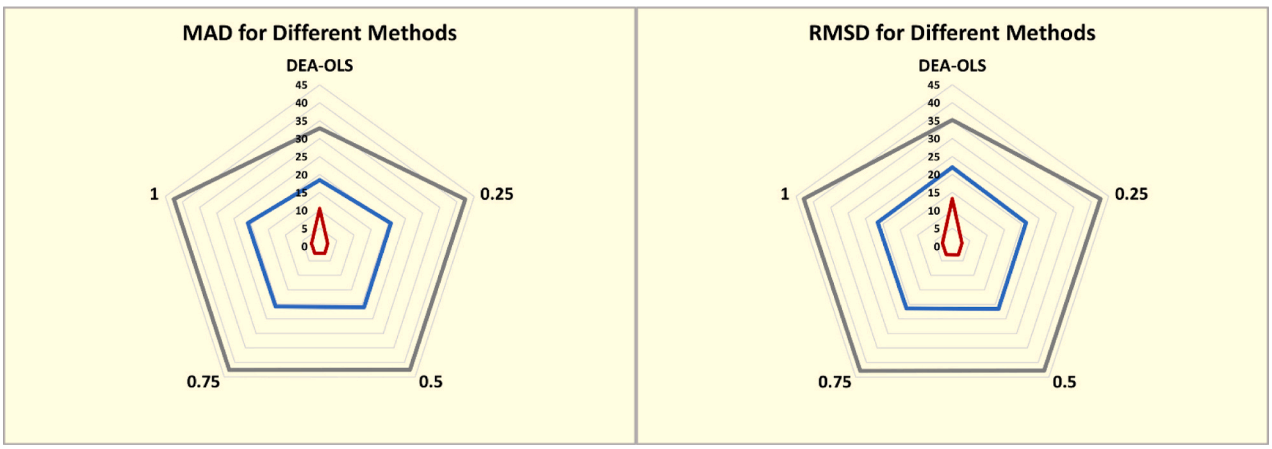
Note that TEA performs better when incorporating a white noise term relative to a deterministic progressive decrease in the effect of technical inefficiencies. In the former scenario, a non-stationary AR model is as accurate as the stationary ones, while in the latter, more persistent shocks worsen the performance of the TEA model. Thus, TEA delivers relatively better outcomes when a small amount of noise is incorporated into the AR model. On the other hand, considering only temporal inefficiency shocks slightly weakens the model's accuracy. In this regard, note that the TEA scenario with $\phi = 1$ is equivalent to DEA-OLS since we simply add a constant term to each observation within each iteration. However, both models improve relative to DEA-OLS. Henceforth, we will focus on the performance of the TEA model absent white noise while highlighting that incorporating a white noise term would lead to relatively better outcomes.

We evaluate the sensitivity of the base TEA ($\varepsilon_t = \phi\varepsilon_{t-1}$) case with $N = 200$ when considering a larger sample size ($N = 400$), a larger impact

Table 1
Comparing TEA ($\varepsilon_t = \phi\varepsilon_{t-1} + e_t$) and DEA-OLS to assess the impact of contextual variables: benchmark sample size.

Estimation method		BCC		FDH (Concave)		FDH (Convex)	
DEA-OLS		MAD	RMSD	MAD	RMSD	MAD	RMSD
		10.58	13.3	18.48	22.02	32.92	35.20
TEA	$\phi = 0.25$	2.33	2.79	20.72	21.24	42.39	42.64
($\varepsilon_t = \phi\varepsilon_{t-1} + e_t$)	$\phi = 0.5$	2.39	2.89	20.96	21.50	42.57	42.82
	$\phi = 0.75$	2.37	2.86	20.76	21.38	42.62	42.89
	$\phi = 1$	2.40	2.86	20.96	21.51	42.51	42.76

Notes: The base case with $\beta = 0{,}2$, σ_u=0,15,$\sigma_\nu = 0{,}04$, $E(e^{-u})$= 0,83, and sample size = 200. Number of iterations performed = 2000.



3(a). MAD Representation **3(b). RMSD Representation**

Fig. 3. A graphical representation of the results in Table 1.

Table 2
Comparing TEA ($\varepsilon_t = \phi\varepsilon_{t-1} + e_t$) and DEA-OLS to assess the impact of contextual variables: small sample size.

Estimation method		BCC		FDH (Concave)		FDH (Convex)	
DEA-OLS		MAD	RMSD	MAD	RMSD	MAD	RMSD
		22.94	28.77	37.51	43.79	61.49	64.80
TEA$\varepsilon_t = \phi\varepsilon_{t-1} + e_t$	$\phi = 0.25$	10.63	12.32	41.13	42.51	74.20	74.73
	$\phi = 0.5$	10.54	12.17	41.05	42.51	73.81	74.34
	$\phi = 0.75$	10.45	12.18	40.84	42.40	74.07	74.61
	$\phi = 1$	10.25	11.85	41.06	42.55	73.72	74.24

Notes: The base case with $\beta = 0{,}2$, σ_u=0,15,$\sigma_\nu = 0{,}04$, $E(e^{-u})$= 0,83, and sample size = 50. Number of iterations performed = 2000.

Table 3
Comparing TEA ($\varepsilon_t = \phi\varepsilon_{t-1}$) and DEA-OLS to assess the impact of contextual variables.

Estimation method		MAD (%)	RMSD (%)
DEA-OLS		10.56	13.10
TEA ($\varepsilon_t = \phi\varepsilon_{t-1}$)	$\phi = 0.25$	3.77	4.79
	$\phi = 0.5$	5.81	7.41
	$\phi = 0.75$	8.12	10.28
	$\phi = 1$	10.50	13.24

Notes: The base case with $\beta = 0{,}2$, σ_u=0,15,$\sigma_\nu = 0{,}04$, $E(e^{-u})$= 0,83, and sample size = 200. The number of iterations performed = 2000.

of the contextual variable on productivity ($\beta = 0.4$), a higher noise level ($\sigma_\nu = 0.1$), and no noise ($\sigma_\nu = 0$). Specifically, we generate 100 sets containing 200 and 400 observations per sample scenario. We further use the base case scenario with 100 sets and 200 observations to analyze the effects of a higher-impact parameter ($\beta = 0.4$), higher noise ($\sigma_\nu = 0.1$), and no noise or measurement error ($\sigma_\nu = 0$). Table 4 shows the performance of TEA in these five scenarios.

We can see from this table that when the sample size increases from

Table 4
Comparison of TEA ($\varepsilon_t = \phi\varepsilon_{t-1}$) performance in five scenarios.

Scenario	Estimation method	MAD (%)	RMSD (%)
Base Case($N = 200$)	$\phi = 0.25$	9.70	10.90
	$\phi = 0.5$	9.90	11.90
	$\phi = 0.75$	11.20	13.60
	$\phi = 1$	12.80	15.80
Large Sample($N = 400$)	$\phi = 0.25$	2.04	2.66
	$\phi = 0.5$	3.49	4.46
	$\phi = 0.75$	5.00	6.37
	$\phi = 1$	6.63	8.36
Larger β($\beta = 0.4$)	$\phi = 0.25$	2.30	2.90
	$\phi = 0.5$	3.20	4.00
	$\phi = 0.75$	4.10	5.30
	$\phi = 1$	5.20	6.70
High Noise($\sigma_{\nu_t} = 0.1$)	$\phi = 0.25$	4.40	5.40
	$\phi = 0.5$	7.20	8.90
	$\phi = 0.75$	10.20	12.40
	$\phi = 1$	13.30	16.20
No Noise($\sigma_{\nu_t} = 0$)	$\phi = 0.25$	3.94	4.72
	$\phi = 0.5$	5.57	6.86
	$\phi = 0.75$	7.41	9.27
	$\phi = 1$	9.39	11.74

Notes: The base case with $\beta = 0{,}2$, σ_u=0,15,$\sigma_\nu = 0{,}04$, $E(e^{-u})$= 0,83, and sample size = 200. Number of iterations performed = 100.

200 to 400 observations, both error metrics, MAD and RMSD decrease significantly. In addition, when the weight of the contextual variable equals 0.4 ($\beta = 0.4$), TEA performs much better than in the base case. These results are similar to those obtained by Banker and Natarajan (2008). Furthermore, in the current framework, the performance of TEA improves relative to the base case when the noise or measurement error increases ($\sigma_{\nu_t} = 0.1$), or vanishes ($\sigma_{\nu_t} = 0$). More precisely, an increase in the relative noise level – namely, an increase in the potential magnitude of the initial shock – enhances the performance of TEA for small autoregressive coefficients. This feature deteriorates as the value of the coefficients increases. A similar trend, but one that displays better TEA performances, follows from a scenario without noise, particularly when the effect of the initial shock vanishes quickly.

3.6. Correlated variables

In this section, we introduce different degrees of serially correlated contextual variables. As noted in Section 2, our proposed three-stage TEA method, as well as the two-stage (DEA-OLS) procedure introduced by Banker and Natarajan (2008), requires independence between the distribution of the vector of inputs $\boldsymbol{X}$ and the vector of contextual variables $\boldsymbol{Z}$ to provide consistent estimators of the contextual variables' impact on productivity. However, the distribution of contextual variables does not need to satisfy the independence requirement. Following Banker and Natarajan (2008), we define two contextual variables, z_1, uniformly distributed on [0, 1], and z_2, which follows from a combination of the first variable with an independently and identically distributed w drawn from a uniform distribution on [0, 1]. The second contextual variable is defined by $z_2 = \rho z_1 + w\sqrt{1-\rho^2}$. It displays a correlation of ρ with the first contextual variable and the same variance.

We consider four different values of $\{0.25, 0.50, 0.75, 1\}$ for the autoregressive parameter and generate 100 sets per correlation and parameter scenario, each consisting of 200 observations. Then, we estimate 100 values for the weights β_1 and β_2 of both contextual variables at nine levels of correlation within the $[-0.8, +0.8]$ range. We compare the values of β_1 and β_2 estimated with the actual values of 0.2 to determine how well TEA performs. Table 5 presents the following MAD (%) and RMSD (%) values for β_1 and β_2 with four different autoregressive parameters under each of the nine correlation scenarios assumed:

$$\text{MAD\%} = 100 \times \frac{1}{0.2}\left(\frac{1}{200}\sum_{t=1}^{100}|\widehat{\beta}_{1t} - 0.2| + |\widehat{\beta}_{2t} - 0.2|\right) \tag{20}$$

$$\text{RMSD\%} = 100 \times \frac{1}{0.2}\left(\frac{1}{200}\sum_{t=1}^{100}(\widehat{\beta}_{1t} - 0.2)^2 + (\widehat{\beta}_{2t} - 0.2)^2\right)^{1/2} \tag{21}$$

The results show how the error metrics of TEA with $\phi = 0.25$ are generally smaller or slightly higher than 20 % for correlation levels between -0.4 and 0.8. Thus, the proposed TEA method remains robust within this parametric scenario when the contextual variables are positively correlated. However, TEA performs substantially worse when the correlation between the contextual variables is high and negative, highlighting the distorting effect that efficiency frictions have on the estimation capacity of the model within this scenario, a feature that should be further analyzed in future research.

We also examine how well TEA – with four different autoregressive parameters – performs when the input variable is correlated with the contextual one. Clearly, the productivity estimates will be biased if we ignore the correlation with the contextual variable when estimating productivity in the second stage – using either parametric or non-parametric methods. As in Wang and Schmidt (2002) and Banker and Natarajan (2008), we run simulations to explore how the biased productivity estimators obtained in the second stage impact the estimates of the contextual variable's weight in the third stage when considering one input and one contextual variable.

The contextual variable z_t is generated by combining two variables; the input variable x_t and a random variable ω_t. The latter is generated using an independent uniform distribution within [0.1]. More precisely, the contextual variable is computed as $z_t = \rho\left\{\frac{x_t - 1}{3}\right\} + \omega_t\sqrt{1-\rho^2}$ with variance equal to that of the random variable, ω_t, and a correlation coefficient between z_t and x_t equal to ρ. As in the previous framework, we generate 100 sets per correlation and parameter scenario, each consisting of 200 observations.

Table 6 illustrates how when the degrees of correlation between the contextual and input variables are between -0.4 and 0.4, MAD (%) does not exceed 21 % for any of the four autoregressive parameters in TEA. As in the previous correlation setting, RMSD (%) performances are inferior to MAD (%) ones. However, in this framework, the performance of TEA weakens when the correlation is relatively high and either positive or negative. Clearly, TEA with $\phi = 0.25$ outperforms the other estimation methods.

All in all, the results obtained illustrate that the performance of TEA and its subsequent benefits are more prominent when the correlation between the contextual and input variables is low or when the correlation between contextual variables is either low or positive.

3.7. Autoregressive input processes

Additional intuition regarding the model's behavior can be obtained by introducing an autoregressive component in the production function. We will assume that DMUs adapt current-period inputs relative to previous periods. We use an AR(1) model in Stage 1 to generate time series data for the input variables, whose domain remains bounded within [1, 4]. Thus, inputs will be assumed to follow a standard AR(1) process, $x_t = \phi x_{t-1} + e_t$, whose expected value, $E[x_{t+h}] = \phi^h x_t$, converges to zero as its length increases and $|\phi| < 1$. To tackle this limit problem, we rescale

Table 5
Comparison of TEA ($\varepsilon_t = \phi\varepsilon_{t-1}$) with four autoregressive parameters for assessing the impact of correlated contextual variables.

Estimation Method		ρ								
		−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8
MAD (%)	$\phi = 0.25$	95.02	95.29	10.24	9.80	9.67	9.83	10.38	11.74	15.15
	$\phi = 0.5$	96.20	96.05	13.95	13.22	13.03	13.28	14.12	16.27	21.49
	$\phi = 0.75$	96.91	96.59	14.16	13.59	13.74	14.33	15.42	17.59	23.21
	$\phi = 1$	93.58	93.57	16.77	15.82	15.72	16.28	17.67	20.59	27.67
RMSD (%)	$\phi = 0.25$	98.68	98.17	17.21	15.82	15.22	15.22	15.94	17.90	23.42
	$\phi = 0.5$	102.16	99.92	22.55	20.92	20.43	20.84	22.30	25.53	33.87
	$\phi = 0.75$	103.19	100.78	20.87	20.04	20.17	21.06	22.88	26.40	34.98
	$\phi = 1$	98.74	97.49	24.03	22.54	22.44	23.44	25.77	30.43	41.84

Notes: The base case assumes that the contextual variables have weights: $\beta_1 = 0.2$, $\beta_2 = 0.2$; $\sigma_u = 0{,}15, \sigma_\nu = 0{,}04$, $E(e^{-u}) = 0{,}83$, and the sample size is 200. Number of iterations performed = 100.

Table 6
Comparison of TEA ($\varepsilon_t = \phi\varepsilon_{t-1}$) with four autoregressive parameters for assessing the impact of correlated contextual and input variables.

Estimation Method		ρ								
		−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8
MAD (%)	$\phi = 0.25$	40.37	26.45	15.61	7.77	3.81	8.61	17.86	29.43	43.28
	$\phi = 0.5$	39.73	26.31	16.09	8.63	4.69	9.28	18.54	30.22	44.15
	$\phi = 0.75$	36.96	24.60	15.41	8.67	5.19	9.77	19.01	30.80	44.91
	$\phi = 1$	37.65	25.63	16.62	9.73	6.08	10.49	20.14	32.36	46.80
RMSD (%)	$\phi = 0.25$	106.45	76.86	50.51	27.21	12.54	25.91	48.18	72.84	100.02
	$\phi = 0.5$	106.70	77.54	51.45	28.26	13.22	25.69	48.04	73.04	100.58
	$\phi = 0.75$	102.44	74.35	49.22	27.04	13.96	26.90	48.85	73.56	100.90
	$\phi = 1$	106.85	78.23	52.47	29.85	16.40	28.29	50.73	76.46	104.88

Notes: The base case assumes that the contextual variable has a weight: β_1= 0.2; the correlation ρ between contextual variables equals 0, σ_u=0,15,σ_ν = 0,04, $E(e^{-u})$= 0,83, and the sample size is 200. Number of iterations performed = 100.

the resulting input realizations within the interval [1,4]. We implement this model through simulations of sizes 50 and 200. In all cases, we assume errors are defined as in Banker and Natarajan (2008).

The results obtained are described in Tables 7 and 8 for simulations of sizes 200 and 50, respectively. Given the AR nature of the inputs, the results are almost identical for all ϕ values, ranging from 0.25 to 1. That is, DEA provides a consistent estimation of efficiency that allows for the identification of the effect of contextual variables when modifying the production function's behavior and adapting input requirements through a time-dependent AR(1) process. This result prevails when considering non-stationary processes displaying a coefficient of one as long as the concavity assumption is maintained. The temporal effect introduced via the production function does not modify the precision of the estimation through the different TEA settings. Therefore, the model validates the results provided by Banker (1993) and Banker and Natarajan (2008) regarding the consistency of the DEA estimator and its capacity to retrieve the true distribution of the error term.

At the same time, Proposition 5 in Banker (1993) requires the realizations of the input variable to be determined by its associated probability density function. Intuitively, the white noise term within the AR(1) process determines the realizations of the input variable after the initial shock is smoothed and assimilated into the production structure of the DMUs. In this regard, the pervasive distortions induced by higher-order autoregressive processes weaken the consistency of the DEA estimator, leading to large MAD values. This trend is reversed when considering a non-stationary process accounting for a constant shock whose prevalence is captured through the white noise term.

4. Case study

This section presents a case study to demonstrate the applicability of TEA methodology in estimating the coefficient of a contextual variable using a real-world economic data set involving 63 countries from 2000 to 2019 (Küçükkocaoğlu & Çakır, 2021). Data availability constraints prevent the use of high-frequency data, a limitation also acknowledged in the conclusion. As a result, the analysis focuses on how the intuition drawn from the numerical results aligns with the theoretical findings presented earlier. While the results may not be fully robust, they offer valuable insights into the contextual effects across different countries, as well as the persistence of inefficiencies and random shocks in their production structures.

Table 7
Comparing TEA ($x_t = \phi x_{t-1} + e_t$) and DEA-OLS to assess the impact of contextual variables: benchmark sample size.

Estimation method		BCC		FDH (Concave)		FDH (Convex)	
DEA-OLS		MAD	RMSD	MAD	RMSD	MAD	RMSD
		10.13	12.74	18.55	22.09	33.03	35.24
TEA ($x_t = \phi x_{t-1} + e_t$)	$\phi = 0.25$	10.25	12.91	18.42	21.76	20.36	23.60
	$\phi = 0.5$	10.61	13.22	19.05	22.47	21.33	24.60
	$\phi = 0.75$	10.59	13.44	20.15	23.51	23.86	26.99
	$\phi = 1$	10.53	13.24	16.59	20.21	28.87	31.72

Notes: The base case with $\beta = 0,2$, σ_u=0,15,σ_ν = 0,04, $E(e^{-u})$= 0,83, and sample size = 200. Number of iterations performed = 2000.

Table 8
Comparing TEA ($x_t = \phi x_{t-1} + e_t$) and DEA-OLS to assess the impact of contextual variables: small sample size.

Estimation method		BCC		FDH (Concave)		FDH (Convex)	
DEA-OLS		MAD	RMSD	MAD	RMSD	MAD	RMSD
		22.88	28.83	37.84	44.29	62.10	65.22
TEA ($x_t = \phi x_{t-1} + e_t$)	$\phi = 0.25$	22.38	27.98	45.97	51.23	49.35	54.20
	$\phi = 0.5$	23.08	29.20	47.99	53.07	52.41	56.89
	$\phi = 0.75$	23.93	30.11	52.91	57.55	61.13	64.74
	$\phi = 1$	22.70	28.33	36.38	42.80	59.43	63.06

Notes: The base case with $\beta = 0,2$, σ_u=0,15,σ_ν = 0,04, $E(e^{-u})$= 0,83, and sample size = 50. Number of iterations performed = 2000.

The current analysis provides insights into the relationship between the rule of law as the contextual variable on the productivity measure of the countries based on Gross Fixed Capital Formation (GFCF) and Gross Domestic Product (GDP) per Capita as input and output variables, respectively. The GFCF variable measures the annual net increase in physical assets within an economy, which includes investments in infrastructure, machinery, buildings, and equipment. This variable reflects the resources allocated to physical capital for production, making it an input. The GDP per Capita is the total economic output of a country divided by its population, which provides an average economic productivity per person, reflecting the standard of living and financial health of a nation. The rule of law ensures that all individuals and institutions are accountable to the law by covering aspects such as legal certainty, property rights, and judicial impartiality. A strong rule of law encourages and enhances investment effectiveness by enforcing contracts, protecting property rights, and fostering a fair economic environment. For further details, we refer the readers to Küçükkocaoğlu and Çakır (2021). Table 9 exhibits the mean of each variable for all countries from 2000 to 2019. The color scale in the table visually represents data variations using different colors.

The scatter plot presented in Fig. 4 shows the relationship between the country averages of x and y. Note that we plot Ln(x) in lieu of x to reduce variation and improve visualization.

The annual mean of x for the 14 developed and 49 developing countries is exhibited in Fig. 5, which shows a clear divergence in the

Table 9
The mean value of variables for 63 countries.

Country	Development Status	GFCF in Millions (x)	GDP Per Capita (y)	Rule of Law	Country	Development Status	GFCF in Millions (x)	GDP Per Capita (y)	Rule of Law
AGO	*	20,600	3115	−1.3105	JAM	*	2,950	4628	−0.337
ALB	*	3,020	3541	−0.567	JPN	**	1,180,000	38,848	1.369
AUS	**	267,000	46,328	1.7735	KAZ	*	29,300	7324	−0.752
BEN	*	1,740	979.5	−0.5165	KEN	*	8,220	1050	−0.75
BFA	*	2,090	529	−0.4615	KGZ	*	1,380	863.2	−1.044
BGD	*	37,700	893	−0.8255	LSO	*	568	1013.8	−0.163
BHR	*	6,310	20,593	0.4615	LUX	**	9,330	95,024	1.8235
BIH	*	2,890	4047	−0.385	MAR	*	24,200	2514	−0.1417
BOL	*	4,010	2085	−0.889	MLT	**	1,780	20,484	1.342
BRA	*	288,000	7950	−0.2065	MNE	*	1,100	5704	0.022
BRD	*	717	15,356	1.114	MNG	*	2,290	2481	−0.208
BWA	*	3,770	5949	0.6125	MOZ	*	2,690	430.2	−0.7315
CHL	*	43,200	11,000	1.282	MRT	*	1,710	1122.8	−0.7425
CHN	*	2,770,000	4814	−0.4505	MYS	*	55,300	8150	0.4705
CIV	*	4,230	1236	−1.081	NER	*	1,810	329.3	−0.605
CMR	*	5,770	1202	−1.1025	NLD	**	156,000	45,773	1.8145
COD	*	5,020	320.1	−1.6545	OMN	*	13,300	15,766	0.4725
COG	*	2,240	2290	−1.2035	PAK	*	27,100	1045.8	−0.821
CYP	**	4,170	26,066	1.0095	POL	*	82,300	10,929	0.614
CZE	**	47,000	16,773	0.9525	RWA	*	1,250	522.9	−0.317
DEU	**	650,000	39,509	1.679	SEN	*	3,490	1139	−0.1635
DNK	**	60,700	52,694	1.9315	SLE	*	459	431.4	−0.975
ECU	*	16,600	4394	−0.9045	SVN	**	9,490	20,805	1.0035
ETH	*	18,600	414.3	−0.698	SWE	**	108,000	48,168	1.9235
FIN	**	52,200	42,620	1.983	TCD	*	2,280	800.1	−1.3545
GEO	*	2,610	2908	−0.254	TGO	*	702	503.9	−0.84
GHA	*	7,250	1572	0.036	TUN	*	8,250	3509	−0.031
GTM	*	6,660	2935	−1.0245	TZA	*	10,200	715.9	−0.415
HTI	*	2,230	633.1	−1.3615	UGA	*	4,360	556.6	−0.4315
IDN	*	189,000	2601	−0.5995	VEN	*	60,400	7433	−1.646
ISL	**	3,590	51,578	1.7755	ZWE	*	1,250	1083.6	−1.5865
ISR	**	56,100	29,659	0.973					

Note: * Developing and ** Developed.

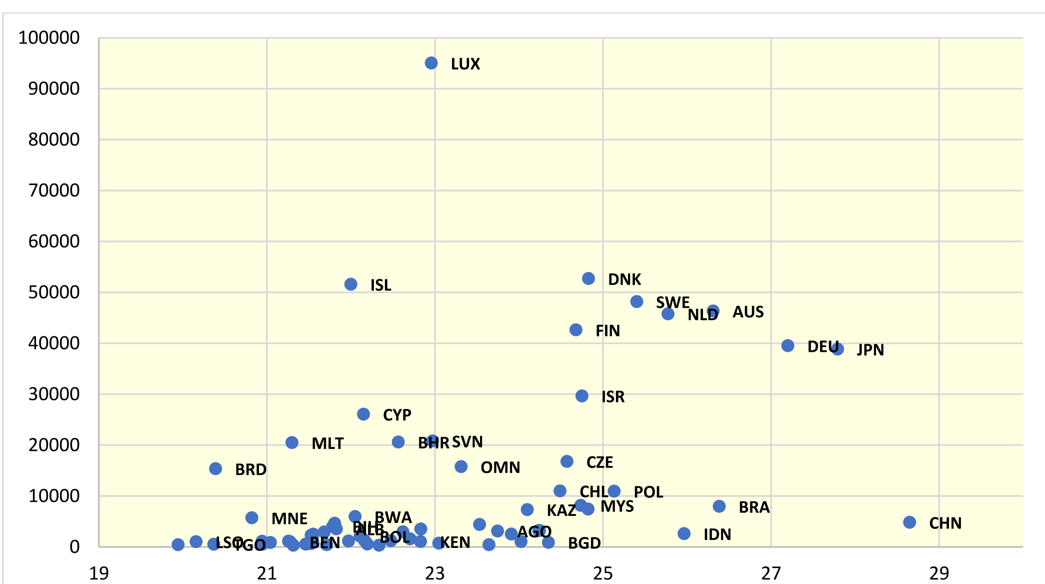


Fig. 4. The scatter plot of Ln(x) versus y.

GFCF mean trends between developed and developing countries. Developed countries exhibit higher GFCF mean values throughout the period, with significant fluctuations. In contrast, the GFCF mean for developing countries remains relatively lower, starting in 2000 and gradually increasing by 10 times by 2019, with less volatility compared to the developed countries. The developing countries have been steadily increasing their investments in physical assets, while the developed countries have experienced more fluctuations in their GFCF mean over the years.

The annual mean of y for the developed and developing countries are shown in Fig. 6. The graph reveals a diverging trend between developed and developing countries, with a notable shift around the 2008 global financial crisis. While developed countries experienced a slowdown in economic output after the 2008 crisis, developing countries were able to maintain a steady increase in their GDP, indicating their resilience and potential for further economic progress.

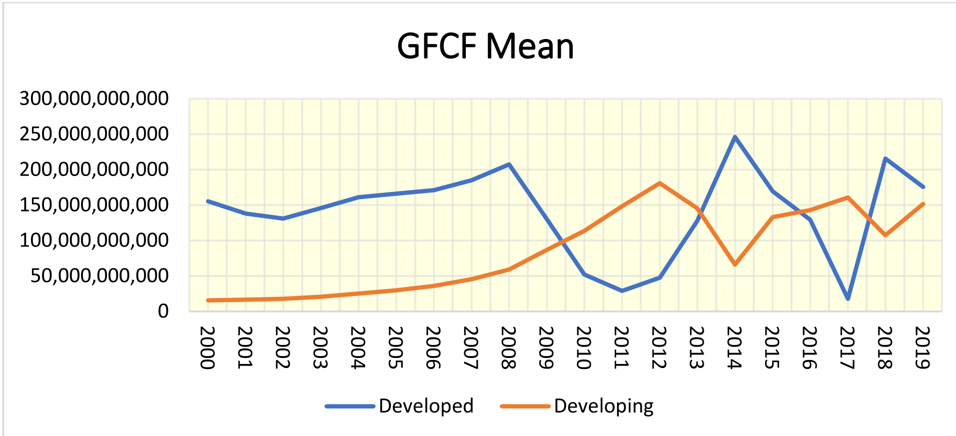


Fig. 5. The annual mean of *x*.

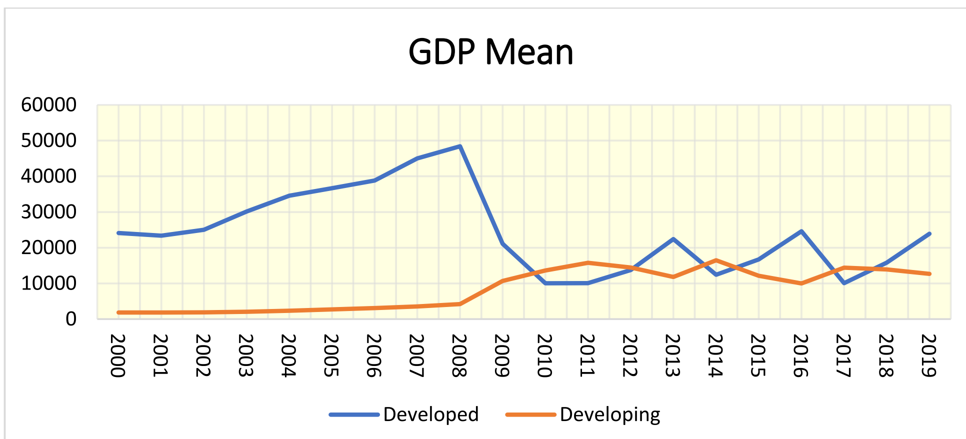


Fig. 6. The annual mean of *y*.

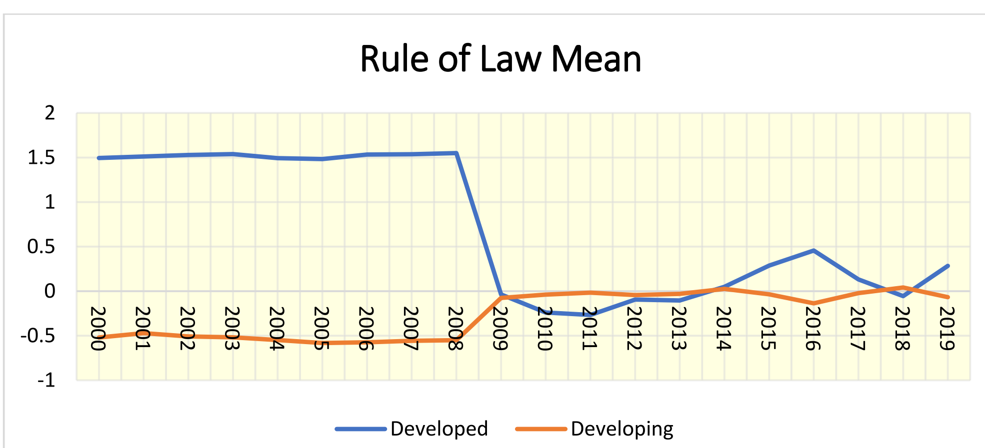


Fig. 7. The annual mean of *z*.

Fig. 7 compares the annual mean of the rule of law for developed and developing countries. There is a clear divergence in the rule of law, which means trends between developed and developing countries, with a notable shift around the 2008 global financial crisis. Before 2008, the mean rule of law of developed countries was significantly higher than that of developing countries, indicating a stronger rule of law and institutional quality in the developed world. However, after the 2008 crisis, the rule of law for developed countries experienced a sharp decline, while the developing countries' rule of law continued to rise, albeit with some fluctuations.

We employ the Prais-Winsten method to estimate the β coefficient in the linear regression model $u_t = \alpha + \beta z_t + \epsilon_t$ for each country where is ϵ_t $AR(1)$. In our analysis, we observed that the first-order autocorrelation coefficients for all countries ranged between 0.98 and 0.99, indicating a strong positive correlation among the error terms. This high level of autocorrelation is statistically significant, suggesting that the errors from the regression model exhibit a persistent structure over time. Furthermore, the second-order autocorrelation coefficients were found to be insignificant across all countries. This absence of significant second-order autocorrelation supports the applicability of the Prais-Winsten method for estimating the β in our linear regression model. The results affirm that our model effectively addresses the first-order autocorrelation while maintaining the assumption of stationary errors, thereby enhancing the reliability of our findings on the impact of z on productivity.

Fig. 8 depicts the $\widetilde{\beta}$ of each country. The countries with the smallest $\widetilde{\beta}$, indicating the weakest relationship between the rule of law and inefficiency, are the Czech Republic, Morocco, Mongolia, Slovenia, Georgia, Bangladesh, and Kyrgyz Republic. These countries have $\widetilde{\beta}$ close to zero and even negative, suggesting that changes in the rule of law have minimal impact on their inefficiency levels. This could be due to other more dominant factors influencing productivity and efficiency in these countries, such as the strength of different institutions, economic policies, or the overall level of development. At the other end of the spectrum, the countries with the largest $\widetilde{\beta}$. Hence, the most substantial relationship between the rule of law and inefficiency is between Ethiopia, Lesotho, Jamaica, the Republic of Congo, and Iceland. These countries exhibit a much stronger connection between the rule of law and inefficiency, with $\widetilde{\beta}$ ranging from 0.472 to 0.904. As illustrated in Table 10, this suggests that improvements in the rule of law in these countries could substantially reduce inefficiency. Conversely, declines in the rule of law could result in significant increases in inefficiency.

Table 10
The estimated β of the countries.

Country	$\widetilde{\beta}$	Country	$\widetilde{\beta}$	Country	$\widetilde{\beta}$
AGO	0.023246	DNK	0.081	MNG	−0.01059
ALB	0.264343	ECU	0.0036	MOZ	0.080589
AUS	0.145183	ETH	0.904474	MRT	0.042943
BEN	0.021651	FIN	0.0504	MYS	0.044589
BFA	0.028183	GEO	−0.01471	NER	0.224897
BGD	−0.01116	GHA	0.219291	NLD	0.17172
BHR	0.141326	GTM	0.048857	OMN	0.216257
BIH	0.10728	HTI	0.009514	PAK	0.207669
BOL	0.053794	IDN	0.030754	POL	0.099257
BRA	0.032451	ISL	0.286971	RWA	0.00648
BRD	0.121834	ISR	0.022731	SEN	0.056263
BWA	0.155417	JAM	0.472731	SLE	0.105583
CHL	0.020777	JPN	0.034046	SVN	−0.02731
CHN	0.169817	KAZ	0.080229	SWE	0.16416
CIV	0.032246	KEN	0.025046	TCD	0.211937
CMR	0.052097	KGZ	−0.02088	TGO	0.003549
COD	0.221451	LSO	0.640851	TUN	0.048497
COG	0.327034	LUX	0.15408	TZA	0.099771
CYP	0.055183	MAR	−0.00273	UGA	0.016817
CZE	−0.00339	MLT	0.065931	VEN	0.009977
DEU	0.056057	MNE	0.053949	ZWE	−0.0055

5. Conclusion and future research directions

DEA allows organizations to compare their performance with the best practices in the industry by estimating the efficiency frontier instead of the production correspondence between the empirical data for the inputs used and the outputs produced. DEA has attracted the interest of an impressive number of researchers and management practitioners worldwide because it provides insights into the efficiency of each DMU analyzed and identifies targets for improving the corresponding

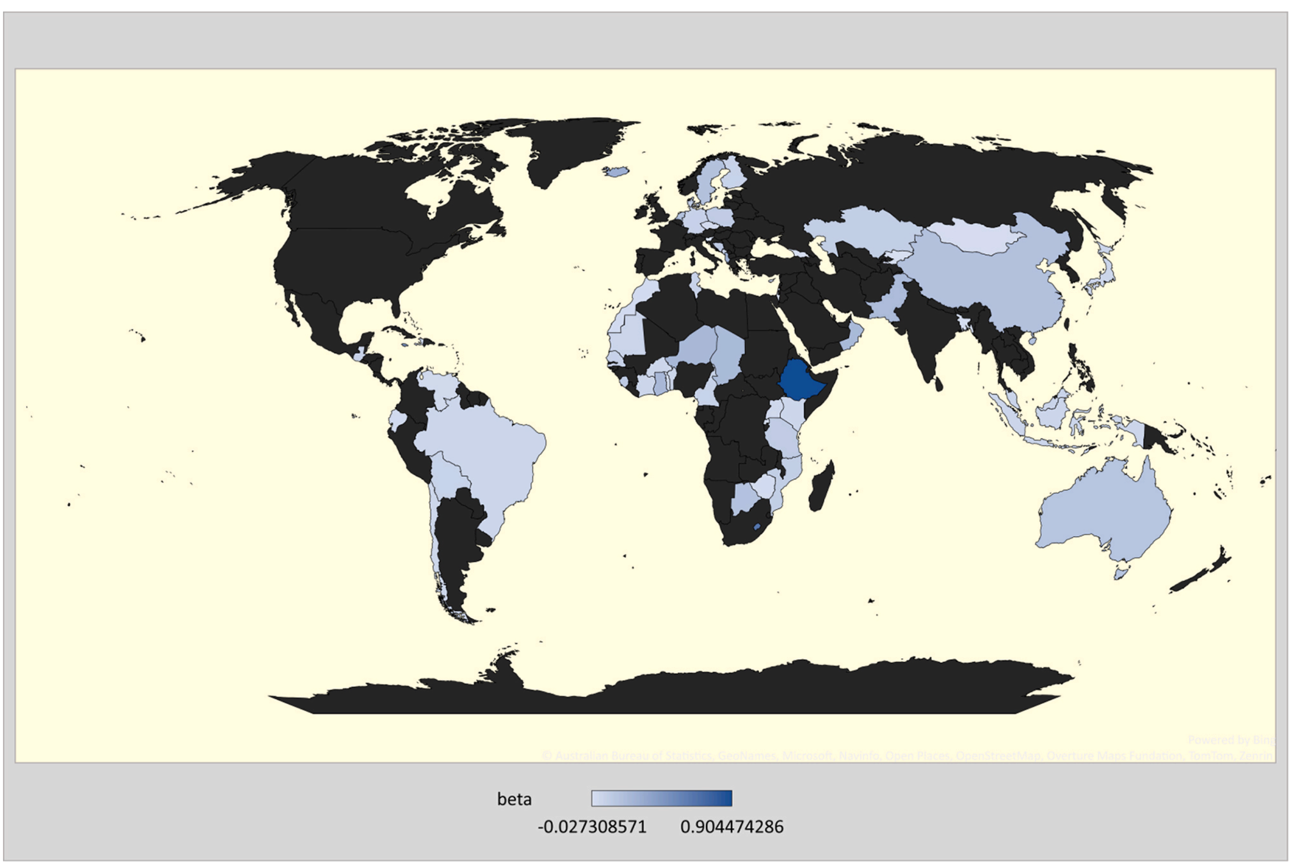


Fig. 8. The MapPlot of the estimated β for the countries.

inefficiencies. It is also important to characterize the drivers of efficiency in organizations and the factors contributing to their improvement. In response, Banker and Natarajan (2008) proposed a two-stage (DEA-OLS) procedure to investigate the impact of contextual variables on productivity and provided simulation evidence illustrating that this procedure significantly outperformed standard parametric methods. Furthermore, Banker et al. (2019) presented evidence showing that OLS in the second stage performs better than truncated regression or bootstrap.

In this study, we have proposed a new DEA-based method in three stages called TEA to effectively incorporate time-series data in DEA. The prevalence of technical inefficiencies and the emergence of random shocks conditioning the productive structure of firms have been incorporated as an AR(1) process into the Banker and Natarajan (2008) framework. We have illustrated their model's capacity to assess contextual variables' impact when the error term follows a structured autoregressive process. Its main components can also be estimated and compared across firms. That is, TEA highlights the ability of the two-stage model of Banker and Natarajan (2008) to incorporate time series data and provide consistent estimations of the productivity of firms and the structural qualities of the random shocks and technical inefficiencies.

We have compared the performance of TEA with the Banker and Natarajan (2008) procedure using four different autoregressive parameters. Our extensive Monte Carlo simulations show that TEA outperforms the two-stage DEA-OLS approach. Furthermore, the accuracy of TEA $(\varepsilon_t = \phi\varepsilon_{t-1})$ improves with smaller autoregressive parameters such as $\phi = 0.25$, while TEA $(\varepsilon_t = \phi\varepsilon_{t-1} + e_t)$ displays higher accuracy independently of the value assigned to the autoregressive parameters. We have also shown that the accuracy of TEA $(\varepsilon_t = \phi\varepsilon_{t-1})$ improves when the correlation between the contextual and input variables is low or when the correlation between contextual variables is either low or positive.

Moreover, we utilized a comprehensive economic dataset from sixty-three countries spanning 2000 to 2019 to examine the impact of the rule of law as the contextual variable on the productivity score of the countries where the input is GFCF and the output is GDP. By fitting a linear regression model of the inefficiency score as the dependent variable and contextual as the predictor, autocorrelated errors were observed ($\in_t AR(1)$). So, the regression coefficient for each country is estimated using the Prais-Winsten method. The differences in the strength of the relationship between the contextual variable and inefficiency score across countries could be attributed to various factors, such as the level of economic development, the quality of other institutions, the historical and cultural context, and the specific challenges faced by each country in establishing and maintaining the rule of law. Countries with weaker relationships may have other institutional or economic factors that overshadow the influence of the rule of law. In comparison, those with stronger relationships may have a more direct and pronounced link between the rule of law and their overall productivity and efficiency.

Clearly, the theoretical framework developed requires additional empirical validation. The daily monitoring of firms allows researchers to retrieve substantial data and analyze the dynamic effects derived from specific contextual factors and the continuous random and efficiency shocks firms face. In this regard, contextual factors can be used to account for the effect of technological modifications on firms' productive capacity. Higher frequency sequential data within the benchmark defined by the productive structure of firms can be retrieved from standard databases such as Refinitiv Eikon (Datastream).

The main drawback regarding the theoretical foundations of TEA is its restriction to AR(1) processes when defining the behavior of the error term variable, with the production structure collapsing whenever the autoregressive coefficient increases. Thus, the main sequential effect triggered by the introduction of time series data is constrained to a lagged period, limiting the range of the persistent structural shocks that can be analyzed through TEA. The model should, therefore, be modified to allow for a more varied set of dynamic scenarios, aiming also to accommodate different time series specifications such as, for instance, MA or ARMA models.

In addition to incorporating time series data, many DEA applications consider panel data. A popular and time-dependent method to analyze panel-series data is Window DEA (WDEA). Peykani et al. (2021) provide a bibliometric analysis of WDEA considering 387 papers published from 1985 to 2020, with 49.4 % published in the last five years of the observation period. Despite the increased interest in WDEA, this method lacks a formal statistical foundation to analyze panel data. Thus, in our future research, we plan to introduce a panel DEA (PDEA) technique that incorporates panel-series data in DEA.

Finally, among the potential extensions of the model, different scenarios that could challenge the main assumptions of TEA should be explored, such as the presence of structural breaks. Similarly, although we have accounted for weakly non-stationary AR processes, the performance effects derived from introducing non-stationary time series should be simulated and evaluated.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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