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Solving multi-period project selection problems with fuzzy goal programming based on TOPSIS and a fuzzy preference relation

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A B S T R A C T

Project portfolio managers are multi-objective Decision-Makers (DMs) who are expected to select the best mix of projects by maximizing profits and minimizing risks over a multi-period planning horizon. However, project portfolio decisions are complex multi-objective problems with a high number of projects from which a subset has to be chosen subject to various constraints and a multitude of priorities and preferences. We propose a Goal Programming (GP) approach for project portfolio selection that embraces conflicting fuzzy goals with imprecise priorities. A fuzzy goal with an aspiration level and a predefined membership function is defined for each objective. The impreciseness in the priorities of the membership values of the fuzzy goals is modeled with fuzzy relations. This leads to type II fuzzy sets since fuzzy relations are organized between the membership values of the fuzzy goals which are themselves fuzzy sets. The proposed model is based on the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and fuzzy preference relations. TOPSIS is used to reduce the multi-objective problem into a bi-objective problem. The resulting bi-objective problem is solved with fuzzy GP (FGP). The fuzzy preference relations are used to help DMs express their preferences with respect to the membership values of the fuzzy goals. The proposed approach is used to solve a real-life problem characterized as a fuzzy Multi-Objective Project Selection with Multi-Period Planning Horizon (MOPS–MPPH). The performance of the proposed approach is compared with a competing method in the literature. We show that our approach generates high-quality solutions with minimal computational efforts.

1. Introduction

Multi-Objective Decision Making (MODM) techniques have attracted a great deal of interest due to their adaptability to real-life decision making problems. In general, solving MODM problems with multiple (often conflicting) objectives requires some form of compromise since the reinforcement of one objective will often worsen some or all of the other conflicting objectives. Therefore, the Decision Maker (DM) must search for a trade-off between the objectives if the best overall result

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is to be obtained. Numerous MODM models have been proposed in the literature for reaching the best compromise between conflicting objectives [34–36,60,77].

Formally, the MODM models involve a vector of decision variables, objective functions, and constraints. Generally, the MODM problem can be formulated as follows:

$$(MODM) \quad \begin{cases} \text{Min} & f(X) \\ \text{S.t.} & X \in S = \{X \in R^n | g(X) \leqslant b,\ X \geqslant 0\} \end{cases} \tag{1}$$

where $f(X)$ represents k different objective functions, $g(X) \leqslant b$ represents m constraints, S is the feasible solution space, and $X \in R^n$ is a vector of decision variables.

The MODM models have been classified into the following four classes according to the preference information: (1) no articulation of the preference information; (2) a priori articulation of the preference information; (3) progressive articulation of the preference information; and (4) posterior articulation of preference information [34,36]. Goal Programming (GP), originally developed by Charnes and Cooper [15], is a mathematical programming technique capable of handling multiple objectives with a priori articulation of the preference information [50]. The preference information in GP is provided as a set of target values (aspiration levels) for the objective functions by the DMs [15]. The key idea behind GP is to minimize the unwanted deviations from the goals set by the DMs [60]. Further development of the original GP model are proposed by Cooper [17], Lee [47], Ignizio [37,38], Hannan [27], Gass [23], Min and Storbeck [51], Jones and Tamiz [40], Romero [56], Romero [57], Liao and Ho [48], and Chang [12]. GP models can be classified into three major subsets (i.e., *non-preemptive*, *lexicographic*, and *Chebyshev*) based on the achievement function used for combining the unwanted deviations [60,57]. Romero [55] presented a comprehensive review of GP models categorized into 18 areas of application and 12 different variants.

Classical GP considers a set of goals with precise and deterministic aspiration levels. However, in real-life problems, there are many decision making situations where the DMs are not able to establish the aspiration levels precisely. A fuzzy GP (FGP) method has been developed to deal with such situations [79,26,10,58,67,20,54,52,59,16,5,13,75,76,31,1,72,6,7,9,49].

In many real-life problems, DMs are simultaneously interested in multidimensional decisions with low-risk and high-return solutions. These decisions become more complicated when the DMs face several conflicting objectives under uncertainty. Considering the DMs' preferences for the priority structure of the aspiration levels makes the applications of MODM more realistic in problems with uncertainties and ambiguities. These problems could be solved by simultaneously modeling the aspiration levels as fuzzy variables and developing a fuzzy relation for the DMs' preferences for the priority of the objectives. In this study, we intend to generate high-quality solutions while modeling the imprecise preferences of the DMs for the priorities of the goals and their aspiration levels. Using the proposed approach, a fuzzy MODM problem is reduced to a bi-objective problem with the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), and high-quality solutions are generated through a simultaneous consideration of low-risk and high-return objectives. All the objectives from the original MODM problem will have an important role in the construction of the resulting bi-objective problem. The resulting bi-objective problem is then solved with the proposed FGP approach. We reduce the computational complexities and take into consideration the uncertain hierarchical structure of the DMs' preferences and the fuzzy priority relations of the membership values of the aspiration levels of the fuzzy goals. More precisely, our proposed approach has two different modules. In the first module, we convert the original MODM problem into a bi-objective problem with TOPSIS. This module seeks solutions that simultaneously have a minimum distance from the Positive Ideal Solution (PIS) and a maximum distance from the Negative Ideal Solution (NIS). In the second module, we propose a new FGP approach to solve the resulting bi-objective problem. The objective function in the final FGP includes a parametric combination of two types of impreciseness. The first type of impreciseness refers to the imprecise preferences for the priority of the membership values of the fuzzy aspiration levels which are modeled with linguistic terms parameterized using a linear fuzzy relation function. The second type of impreciseness refers to a weighted sum of the membership values of the fuzzy aspiration levels. The parametric nature of the proposed approach helps DMs generate solutions with desirable trade-offs between the objectives in a fuzzy environment.

The overall properties of the proposed procedure (i.e., confining the objective dimension space while considering both the ideal and the anti-ideal situations using TOPSIS, and extending the linear fuzzy relation between the membership values of the aspiration levels for different fuzzy goals while interacting with the DMs) make it robust and well-posed for modeling real-life problems. It is worth noting that a Multi-Objective Project Selection with Multi-Period Planning Horizon (MOPS–MPPH) in a fuzzy environment has also been developed. The proposed procedure and the extended version of the FGP method proposed by Aköz and Petrovic [5] are applied to the MOPS–MPPH.

The remainder of the paper is organized as follows. In Section 2 we present the application of the TOPSIS method for solving MODM problems. The proposed approach including a MODM problem using TOPSIS and an extended version of the fuzzy priority relation are described in Section 3. In Section 4 we develop the MOPS–MPPH proposed in this study. The proposed approach and an extended version of the FGP procedure developed by Aköz and Petrovic [5] are applied to the MOPS–MPPH in Section 5. Finally, we end the paper with our conclusions and future research directions in Section 6.

2. TOPSIS method for solving the MODM problems

The TOPSIS method, introduced by Hwang and Yoon [35], is a well-known MODM approach. A wide variety of TOPSIS applications have been reported in the MADM literature. Kahraman et al. [41] proposed a decision process for selecting new product ideas based on fuzzy heuristic multi-attribute utility and hierarchical fuzzy TOPSIS. Abo-Sinna and Amer [2] and Abo-Sinna et al. [4] used TOPSIS to solve multi-objective large-scale non-linear programming problems with block angular structure. Abo-Sinna and Abou-El-Enien [3] applied TOPSIS to large-scale multiple objective programming problems involving fuzzy parameters. Jadidi et al. [39] used and extended the version of the TOPSIS method proposed by Abo-Sinna and Abou-El-Enien [3] to solve the multi-objective supplier selection problem under price breaks using multi-objective mixed integer linear programming. Lai et al. [46] used the compromise properties of TOPSIS to generate solutions with the shortest distance from the PIS as well as the longest distance from the NIS and reducing a k-dimensional objective space to a two-dimensional objective space by a first-order compromise procedure. Recently, Khalili-Damghani et al. [45] used a TOPSIS method to confine the objective dimension space of real-life large-scale multi-objective multi-period project selection problems.

2.1. Algorithm I

In this section we briefly describe the TOPSIS method for MODM:

Step 1. Considering the original multiple objective optimization problem with k conflicting objectives as (2), we solve two sets of single objective optimization problems as (3) and (4):

$$\{\text{Optimize } f_i(X),\ i = 1,2,\ldots,k;\ g_j(X) \leqslant b,\ j = 1,2,\ldots,m.\} \tag{2}$$
$$\{\text{Min } f_i(X),\ i = 1,2,\ldots,k;\ g_j(X) \leqslant b,\ j = 1,2,\ldots,m.\} \tag{3}$$
$$\{\text{Max } f_i(X),\ i = 1,2,\ldots,k;\ g_j(X) \leqslant b,\ j = 1,2,\ldots,m.\} \tag{4}$$

where m and k have the same definitions as in *Problem (1)*, $g_j(X) \leqslant b, j = 1,2,\ldots,m$ are the same set of constraints in *Problem (1)*, and b is a constant.

Step 2. Form the pay-off table for the objective functions and calculate Z^+, and Z^- as follows:

$$Z^- = \left(Z_1^-, Z_2^-, \ldots, Z_i^-, \ldots, Z_{k-1}^-, Z_k^-\right) \tag{5}$$
$$Z^+ = \left(Z_1^+, Z_2^+, \ldots, Z_i^+, \ldots, Z_{k-1}^+, Z_k^+\right) \tag{6}$$

where Z^- is a vector of the optimum values of the single objective problem which has been optimized in the contrary direction of the original MODM problem (i.e., NIS). On the other hand, if $f_1(X)$ is to be minimized in the original Problem (1), then Z_1^- is the optimum value of the problem $\{\text{Max } f_1(X);\ g_j(X) \leqslant b, j = 1,2,\ldots,m\}$, (i.e., $Z_1^- = f_1^*(X)$) and Z_1^+ is the optimum value of the problem $\{\text{Min } f_1(X);\ g_j(X) \leqslant b, j = 1,2,\ldots,m.\}$, (i.e., $Z_1^+ = f_1^*(X)$).

Z^+ is a vector of optimum values of the single objective problem which has been optimized in the same direction of the original MODM problem (i.e., PIS). On the other hand, if $f_1(X)$ is to be maximized in the original Problem (1), then, Z_1^+ is the optimum value of the problem $\{\text{Max } f_1(X);\ g_j(X) \leqslant b, j = 1,2,\ldots,m.\}$, (i.e., $Z_1^+ = f_1^*(X)$) and Z_1^- is the optimum value of the problem $\{\text{Min } f_1(X);\ g_j(X) \leqslant b, j = 1,2,\ldots,m.\}$, (i.e., $Z_1^- = f_1^*(X)$).

It is clear that the range of the objective functions which are maximized in the original MODM problem (i.e., $Z_i^+ > Z_i^-, i = 1,2,\ldots,k$), can be estimated by $Z_i^+ - Z_i^-, i = 1,2,\ldots,k$. In contrast, the range of the objective functions which are minimized in the original MODM problem (i.e., $Z_i^+ < Z_i^-, i = 1,2,\ldots,k$), can be estimated by $Z_i^- - Z_i^+, i = 1,2,\ldots,k$.

Step 3. Using the NIS, the PIS, the range of the objective functions, the DMs' opinions about the relative importance of the objective functions, and considering i, ($i = 1,2,\ldots,k$) different objective functions in the original MODM problem (which have been divided into k_1 minimizing objective functions and $k - k_1$ maximizing objective functions), we calculate the distance function from the PIS (i.e., $d_p^{PIS}(x)$) and the distance function from the NIS (i.e. $d_p^{NIS}(x)$) as follows:

$$d_p^{PIS}(x) = \left[\sum_{i=1}^{k_1}\left[W_i \times \frac{(f_i(X) - z_i^+)}{z_i^- - z_i^+}\right]^p + \sum_{i=k_1+1}^{k}\left[W_i \times \frac{(z_i^+ - f_i(X))}{z_i^+ - z_i^-}\right]^p\right]^{\frac{1}{p}} \tag{7}$$

$$d_p^{NIS}(x) = \left[\sum_{i=1}^{k_1}\left[W_i \times \frac{(z_i^- - f_i(X))}{z_i^- - z_i^+}\right]^p + \sum_{i=k_1+1}^{k}\left[W_i \times \frac{(f_i(X) - z_i^-)}{z_i^+ - z_i^-}\right]^p\right]^{\frac{1}{p}} \tag{8}$$

The parameter w_i in Eqs. (7) and (8) reflects the importance weight of the distance of each objective function from its PIS and NIS in the original MODM problem, respectively. The sum of w_i over the index i is equal to unit. The parameter p is a positive integer ($p \in \{1,2,3,\ldots\} \cup \{\infty\}$) used to control the compromise solution in TOPSIS. The distance decreases as p increases. More specifically, $p = 1$ refers to the *Manhattan* distance (the longest distance in the geometrical sense), $p = 2$, refers to the *Euclidean* distance (the shortest distance in the geometrical sense), and $p = \infty$, refers to the *Tchebycheff* distance (the shortest distance in the numerical sense). It is notable that $d_p^{PIS}(x)$ and $d_p^{NIS}(x)$ are scale independent since they are normalized to get values in the range [0,1]. Before normalization, it is not possible to compare objectives with different units of measurement. However, after normalization comparing them is possible because their values are independent of the scale and the metrics of the objective functions in the original MODM problem. Although $d_p^{PIS}(x)$ and $d_p^{NIS}(x)$ are scale independent, they contain all the relevant measurement information for the objective functions in the original MODM problem.

Step 4. Given the overall goal of generating solutions which simultaneously have a minimum distance from the PIS and a maximum distance from the NIS, we develop the bi-objective problem (9)–(11) as follows:

$$\text{Min} \quad d_p^{PIS}(x) \tag{9}$$

$$\text{Max} \quad d_p^{NIS}(x) \tag{10}$$

$$\text{S.t.} \quad x \in S \tag{11}$$

The resulting bi-objective problem can be solved using one of the aforementioned existing procedures in the literature [2–4,39]. We refer to Eqs. (9)–(11) as the *TOPSIS-based bi-objective problem* where S is the feasible solution space in the original MODM problem. In other words, the expression $x \in S$ means that all the constraints of the general models (2)–(4) are satisfied (i.e., $g_j(X) \leqslant b$, $j = 1,2,\ldots,m$).

3. Proposed approach

In this section we develop a parametric MODM procedure based on TOPSIS with an uncertain hierarchical structure of the DMs' preferences on the membership values of the fuzzy aspiration levels.

3.1. Presentation of the DM's preferences

Fuzzy relations are effective paradigms for representing DMs' preferences with respect to different aspects of optimization problems [42,43,64,44,21,18,19].

Definition 3.1. A fuzzy relation is a fuzzy set defined in a Cartesian product of crisp sets $U_1, U_2, \ldots, U_n$. More formally, a fuzzy relation R in $U_1 \times U_2 \times \cdots \times U_n$ is defined as the fuzzy set $R = \{((U_1, U_2, \ldots, U_n),\ \mu_R(U_1, U_2, \ldots, U_n)) | (U_1,\ U_2, \ldots, U_n) \in U_1 \times U_2 \times \cdots \times U_n\}$ where μ_R: $U_1 \times U_2 \times \cdots \times U_n \to [0,1]$.

Chiclana et al. [18,19] introduced different forms of representation for the DMs' preferences over a set of alternatives. They also proposed an integration procedure for different combination of DMs' preference formats.

Suppose $X = \{x_1, \ldots, x_n\}$, $n \geqslant 2$ is a finite set of alternatives. The alternatives are classified from best to worst according to the DMs' preferences. The DMs' preferences for a set of alternatives, X, can be represented according to one of the following definitions:

Definition 3.2. An individual preference ordering on X is defined as $O = \{o(1), \ldots, (o(n)\}$, where $o(\cdot)$ is a permutation function over the index set $\{1, \ldots, n\}$.

Definition 3.3. A fuzzy preference relation on X is described through $P \subset X \times X$, with a membership function, μ_P: $X \times X \to [0,1]$, where $\mu_P(x_i, x_j) = p_{ij}$ denotes the preference degree of alternative x_i over x_j.

Definition 3.4. A utility preference form on X is defined as a set of n utility values, $U = \{u_i,\ i = 1, \ldots, n\}$, $u_i \in [0,1]$, where u_i represents the utility evaluation of the DM to alternative x_i.

Definition 3.5. A positive preference relation on X is defined through $A \subset X \times X$, $A = [a_{ij}]$, where a_{ij} indicates a ratio of the preference intensity for alternative x_i to that of x_j.

Szmidt and Kacprzyk [62,63] used the intuitionistic fuzzy preference relation to study the consensus-reaching process, and to analyze the extent of agreement in a group of experts. Xu [74] developed a special variation of GP for obtaining the priority vector of incomplete fuzzy preference relations. Fan et al. [21] proposed a method for solving the MODM problems based on a linear GP model, where the preferences of the DM on the alternatives were represented with a fuzzy relation. Fan et al. [22] also proposed a GP model for solving group decision-making problems where the preference information on the alternatives provided by the DMs was represented with multiplicative preference relations and fuzzy preference relations. Wang and Fan [69] proposed a chi-square method for obtaining a priority vector from an arbitrary mixture of the multiplicative and fuzzy preference relations of the DMs on the alternatives. Wang and Chen [70] used fuzzy linguistic preference relations for constructing a pairwise comparison matrix in the fuzzy analytical hierarchy process method with additive reciprocal property and consistency. Wang et al. [71] presented an optimization aggregation approach for determining the relative weights of individual fuzzy preference relations. Chang and Wang [14] used the consistent fuzzy preference relations for a forecasting procedure concerning the success of advanced manufacturing technology implementation based on seven influential factors. The method reduced the ratio of the pairwise comparisons of the priority weights in comparison with the analytic hierarchy process. Gong et al. [24] proposed GP models for deriving the priority vector of the intuitionistic fuzzy preference relations.

3.2. Extension of the uncertain priority of the objectives using fuzzy relations

The main challenge in FGP is to represent the hierarchical structure of a DM's preferences on the fuzzy priority of the goals and their associated fuzzy aspiration levels. The fuzzy relation between the membership values of the fuzzy goals has been effectively used to represent uncertain hierarchical structures of the DMs' preferences on the priority of the membership values of the fuzzy goals. Considering Definition 3.3 and customizing it for *Problem (1)* results in Definitions 3.6 and 3.7 as follows:

Definition 3.6. Let $G = \{\tilde{g}_1, \ldots, \tilde{g}_n\}$ be a finite set of fuzzy goals and $X = \{x_1, \ldots, x_m\}$ be the set of decision variables in *Problem (1).* The membership of the fuzzy goal s, is $\mu_s(X)$: $R^n \to [0,1]$, $i = 1, \ldots, n$.

Definition 3.7. Let $G = \{\tilde{g}_1, \ldots, \tilde{g}_n\}$ be a finite set of fuzzy goals, $X = \{x_1, \ldots, x_m\}$be the set of decision variables in *Problem (1)*, and $F = \{\mu_1(X), \ldots, \mu_n(X)\}$ be a finite set of membership values of the fuzzy goals in G. The DMs' preferences on F is represented with a fuzzy preference relation, $R \subset F \times F$, with membership function, μ_R: $F \times F \to [0,1]$ where $\mu_R(\mu_s(X), \mu_t(X))$ denotes the membership value of the preference of the membership value of the fuzzy goal*s* (i.e., $\mu_s(X)$) over the membership value of the fuzzy goal t (i.e., $\mu_t(X)$).

It can be concluded that R is a function of the membership value of the fuzzy goals while the membership value of the fuzzy goals are a function of the decision variables, X, themselves. Formally, we organize a fuzzy relation based on several membership functions of fuzzy goals, all considered simultaneously in a FGP. First, the fuzzy sets adjust the decision variables and the membership values of the fuzzy goals through $\mu_s(X)$ for a given fuzzy goal s. The hierarchical structure for the DM's preferences on the membership values of the fuzzy goals is then modeled through fuzzy relation R. This type of uncertainty modeling can be assumed as a real-life application of the type II fuzzy sets in which a fuzzy relation has been organized between the memberships values of the fuzzy goals which are fuzzy sets themselves. Clearly, the binary value of μ_R for a DM's preferences on the membership values of the two given fuzzy goals is not useful. In other words, different values for the DM's preferences on a given set of membership values of the fuzzy goals should be supplied through a suitable function of $\mu_s(X)$ and $\mu_t(X)$. Therefore, $\mu_s(X)–\mu_t(X)$ can be a suitable function for representing this type of combined relation. It is worth noting that $\mu_s(X)$ is abbreviated to μ_s in favor of simplicity throughout the remaining parts of the manuscript. More formally, μ_R which represents the preference of μ_s over μ_t is followed by R_q, $q = 1,2, \ldots, 10$, which is a function of $\mu_s - \mu_t$.

One of the main features of the MODM procedures is their ability in handling the DMs' preferences on the priority of the goals. The hierarchical structure of the DMs' preferences on the priorities of the membership values of the fuzzy goals could be represented through fuzzy relations. The DMs can determine their preferences for the priority relations of the membership values of the fuzzy goals in the form of linguistic terms which can be parameterized with fuzzy relations. Herrera et al. [28–30] have proposed a framework based on the linguistic decision processes for group decision making problems. Generally, the value for a variable can be represented by words in natural languages. This representation through linguistic variables is characterized by fuzzy sets defined in the universe of discourse in which the variable is defined.

We propose a framework for capturing the DM's preferences on the membership values of a fuzzy goal through 3 different linguistic terms subdivided into 10 different linguistic sub-terms. Each sub-term is associated with a fuzzy preference relation which is modeled with a linear fuzzy membership function. More formally, we develop a procedure using a wide range of linguistic terms, different fuzzy priority relations for the DMs' preferences on the membership values of the fuzzy goals, and their associated membership functions. The approach framework is depicted in Fig. 1 while the details of the linguistic sub-terms and their associated fuzzy relation membership functions are shown in Table 1 and Fig. 2, respectively. Without loss of generality, we have assumed that the fuzzy preference relations are represented with linear membership functions.

As the DM's preference on the priority of the membership values of two given fuzzy goals tends toward "*extremely more important*" (i.e., toward R_9 in Table 1), the membership values (i.e., μ_R) of a given member (i.e., $\mu_s–\mu_t$) in the set will be smaller. *Incomparability* (i.e., case (j) in Fig. 2) and *Exactly Equal* (i.e., case (a) in Fig. 2) can be handled with the existing crisp methods in the literature. Case (j) in Fig. 2 shows that goal s is unanimously preferred to goal t. Therefore, $\mu_{R_{10}}$ will have a positive value equal to 1 if and only if $\mu_s–\mu_t$ is equal to 1; otherwise, it will be 0. Case (a) in Fig. 2 shows the indifference between goal s and goal t. Therefore, μ_{R1} will achieve a positive value equal to 1 if and only if $\mu_s–\mu_t$ is equal to 0; otherwise, it will be 0. All the remaining cases in Fig. 2, which are modeled through linguistic terms parameterized with fuzzy relations, denote that the membership value of goal s is preferred to the membership value of goal t to some extent.

The membership function of the fuzzy relations between the fuzzy goals can be formulated as follows:

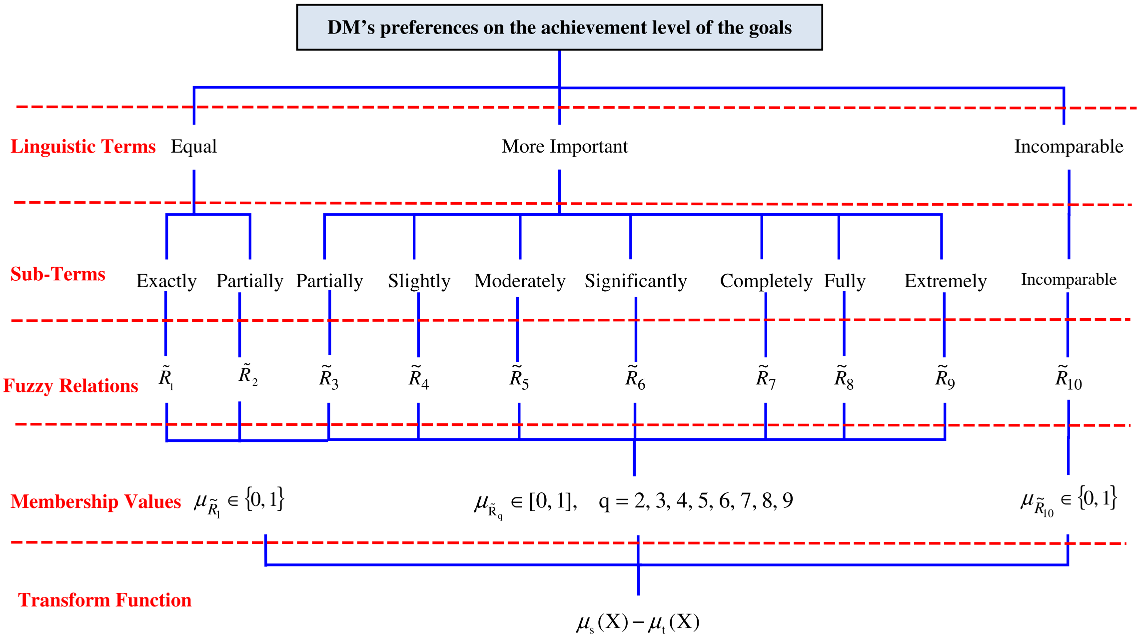


Fig. 1. The hierarchical structure of the DM's preferences on the achievement level of the goals.

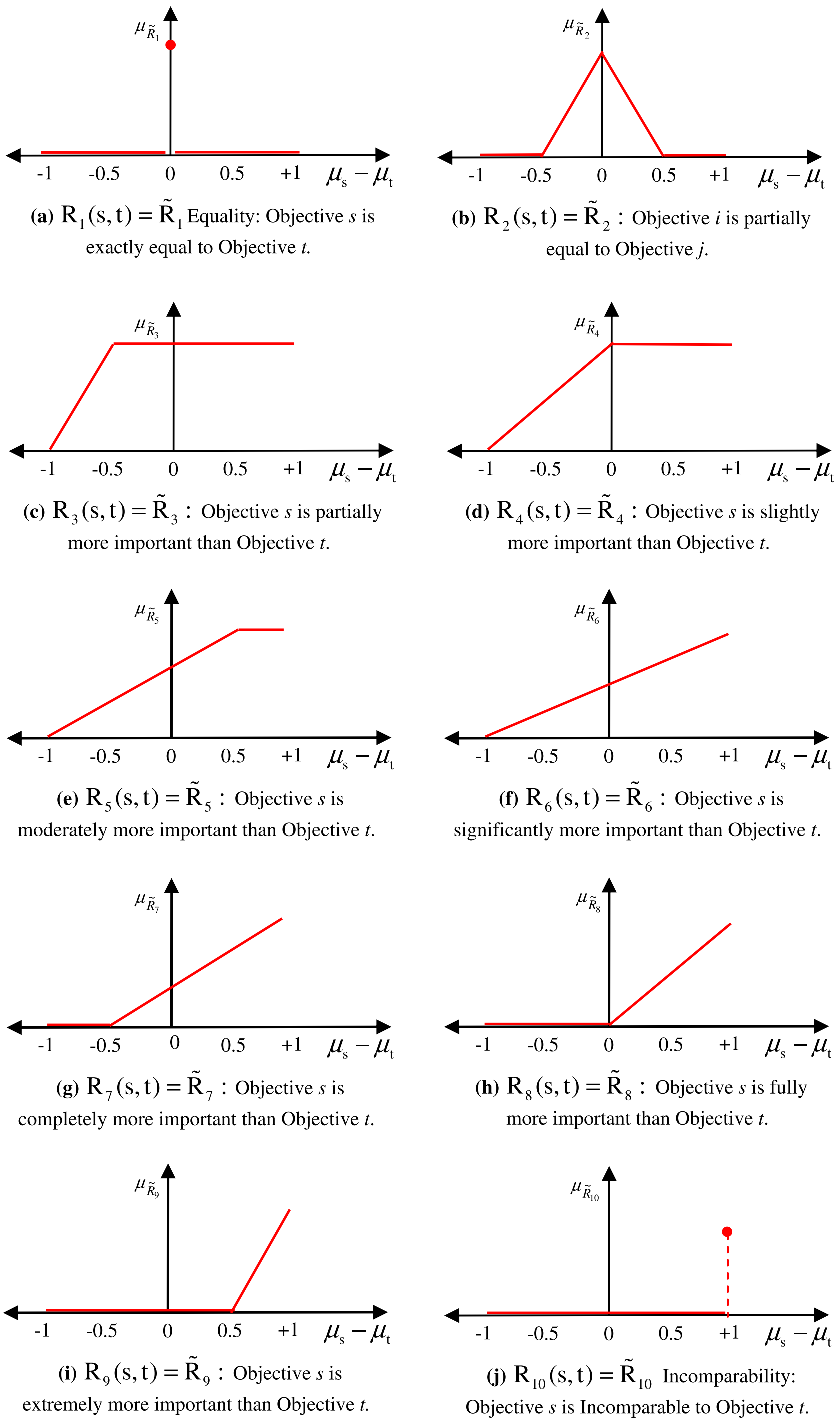
Table 1
Linguistic terms and their associated fuzzy relations.

Linguistic term	Linguistic sub-term	Fuzzy relation
Equal	Exactly equal	$\tilde{R}_1$
	Partially equal	$\tilde{R}_2$
More important	Partially more important than	$\tilde{R}_3$
	Slightly more important than	$\tilde{R}_4$
	Moderately more important than	$\tilde{R}_5$
	Significantly more important than	$\tilde{R}_6$
	Completely more important than	$\tilde{R}_7$
	Fully more important than	$\tilde{R}_8$
	Extremely more important than	$\tilde{R}_9$
Incomparable	Incomparable	$\tilde{R}_{10}$

$$\mu_{\tilde{R}_1} = \begin{cases} 0 & \text{if } -1 \leqslant \mu_s - \mu_t < 0 \\ 1 & \text{if } \mu_s - \mu_t = 0 \\ 0 & \text{if } 0 < \mu_s - \mu_t \leqslant +1 \end{cases} \tag{12}$$

$$\mu_{\tilde{R}_2} = \begin{cases} 0 & \text{if } -1 \leqslant \mu_s - \mu_t \leqslant -0.5 \\ 2(\mu_s - \mu_t + 0.5) & \text{if } -0.5 \leqslant \mu_s - \mu_t \leqslant 0 \\ -2(\mu_s - \mu_t - 0.5) & \text{if } 0 \leqslant \mu_s - \mu_t \leqslant 0.5 \\ 0 & \text{if } 0.5 \leqslant \mu_s - \mu_t \leqslant +1 \end{cases} \tag{13}$$

$$\mu_{\tilde{R}_3} = \begin{cases} 2(\mu_s - \mu_t + 1), & \text{if } -1 \leqslant \mu_s - \mu_t \leqslant -0.5 \\ 1 & \text{if } -0.5 \leqslant \mu_s - \mu_t \leqslant +1 \end{cases} \tag{14}$$



(a) $R_1(s,t) = \tilde{R}_1$ Equality: Objective *s* is exactly equal to Objective *t*.

(b) $R_2(s,t) = \tilde{R}_2$: Objective *i* is partially equal to Objective *j*.

(c) $R_3(s,t) = \tilde{R}_3$: Objective *s* is partially more important than Objective *t*.

(d) $R_4(s,t) = \tilde{R}_4$: Objective *s* is slightly more important than Objective *t*.

(e) $R_5(s,t) = \tilde{R}_5$: Objective *s* is moderately more important than Objective *t*.

(f) $R_6(s,t) = \tilde{R}_6$: Objective *s* is significantly more important than Objective *t*.

(g) $R_7(s,t) = \tilde{R}_7$: Objective *s* is completely more important than Objective *t*.

(h) $R_8(s,t) = \tilde{R}_8$: Objective *s* is fully more important than Objective *t*.

(i) $R_9(s,t) = \tilde{R}_9$: Objective *s* is extremely more important than Objective *t*.

(j) $R_{10}(s,t) = \tilde{R}_{10}$ Incomparability: Objective *s* is Incomparable to Objective *t*.

Fig. 2. Linear fuzzy relation membership functions.

$$\mu_{\tilde{R}_4} = \begin{cases} (\mu_s - \mu_t + 1), & \text{if } -1 \leqslant \mu_s - \mu_t \leqslant 0 \\ 1 & \text{if } 0 \leqslant \mu_s - \mu_t \leqslant +1 \end{cases} \tag{15}$$

$$\mu_{\tilde{R}_5} = \begin{cases} \frac{2}{3}(\mu_s - \mu_t + 1), & \text{if } -1 \leqslant \mu_s - \mu_t \leqslant 0.5 \\ 1 & \text{if } 0.5 \leqslant \mu_s - \mu_t \leqslant +1 \end{cases} \tag{16}$$

$$\mu_{\tilde{R}_6} = \frac{(\mu_s - \mu_t + 1)}{2}, \quad \text{if } -1 \leqslant \mu_s - \mu_t \leqslant +1 \tag{17}$$

$$\mu_{\tilde{R}_7} = \begin{cases} 0, & \text{if } -1 \leqslant \mu_s - \mu_t \leqslant -0.5 \\ \frac{2}{3}(\mu_s - \mu_t + 0.5), & \text{if } -0.5 \leqslant \mu_s - \mu_t \leqslant +1 \end{cases} \tag{18}$$

$$\mu_{\tilde{R}_8} = \begin{cases} 0, & \text{if } -1 \leqslant \mu_s - \mu_t \leqslant 0 \\ (\mu_s - \mu_t), & \text{if } 0 \leqslant \mu_s - \mu_t \leqslant +1 \end{cases} \tag{19}$$

$$\mu_{\tilde{R}_9} = \begin{cases} 0, & \text{if } -1 \leqslant \mu_s - \mu_t \leqslant +0.5 \\ 2(\mu_s - \mu_t - 0.5), & \text{if } +0.5 \leqslant \mu_s - \mu_t \leqslant +1 \end{cases} \tag{20}$$

$$\mu_{\tilde{R}_{10}} = \begin{cases} 0, & \text{if } -1 \leqslant \mu_s - \mu_t < +1 \\ 1 & \text{if } \mu_s - \mu_t = +1 \end{cases} \tag{21}$$

where s and $t(s \neq t)$ are indices of two arbitrary objective functions in the original MODM problem.

3.3. Proposed fuzzy mathematical programming

Considering the *TOPSIS-based bi-objective problem* (9)–(11), the extended fuzzy relation developed in (12)–(21), and the FGP model proposed by Tiwari et al. [68,69], the following model (called *TOPSIS-based goal programming with fuzzy priorities*) is proposed for solving the MODM problems. The parameters of model (22)–(43) are defined as follows:

λ, $0 \leqslant \lambda \leqslant 1$	The parameter assigned to fine-tune convex combination of the weighted additive achievement degrees of the fuzzy goals and the weighted sum of the DM's preferences on the membership values of the fuzzy goals
μ_p^{PIS}	The membership value of the goal of the first objective in model (9)–(11)
μ_p^{NIS}	The membership value of the goal of the second objective in model (9)–(11)
w_p^{PIS}	The relative importance of the satisfaction level of the first fuzzy goal in model (9)–(11)
w_p^{NIS}	The relative importance of the satisfaction levels of the second fuzzy goal in model (9)–(11)
l_{st}, $s, t \in \{1,2\}$, $s \neq t$	A binary variable which is equal to 1 if an importance relation has been defined between the membership values of $d_p^{PIS}(x)$ and $d_p^{NIS}(x)$; it is equal to 0 otherwise
w_{st}, $0 \leqslant w_{st} \leqslant 1$	The relative importance of the DM's preference on the satisfaction levels of the fuzzy goals s and fuzzy goals t
$\widetilde{R}_k(s,t) = \widetilde{R}_q, q = 1,2,\ldots,10$	The fuzzy relation type k defined between the satisfaction levels of the fuzzy goals of the objective functions in model (9)–(11) (i.e., $d_p^{PIS}(x)$ and $d_p^{NIS}(x)$)
$\mu_{\widetilde{R}_k(s,t)}, k = 1,2,\ldots,10$	The membership value of the fuzzy relation type k defined between the membership values of $d_p^{PIS}(x)$ and $d_p^{NIS}(x)$
d_p^{PIS+}	The minimum distance from the PIS for the compromise ratio of p when model (9)–(11) is solved as a single-objective problem
d_p^{PIS-}	The maximum distance from the PIS for the compromise ratio of p when model (9)–(11) is solved as a single-objective problem
d_p^{NIS-}	The minimum distance from the NIS for the compromise ratio of p when model (9)–(11) is solved as a single-objective problem
d_p^{NIS+}	The maximum distance from the NIS for the compromise ratio of p when model (9)–(11) is solved as a single-objective problem
y	A binary variable that enforces just one of the constraints (26) and (27) to be active simultaneously
M	A very large constant value
X	The vector of the decision variables in the original MODM problem
S	The feasible solution space in the original MODM problem

$$\text{Max} \quad \lambda \times \left(w_p^{PIS} \times \mu_p^{PIS} + w_p^{NIS} \times \mu_p^{NIS}\right) + (1-\lambda) \times \sum_{s=1}^{2} \sum_{\substack{t=1 \\ t \neq s}}^{2} (w_{st} l_{st} \mu_{\widetilde{R}(s,t)}) \tag{22}$$

$$\text{S.t.} \quad \mu_p^{PIS} \leqslant \frac{d_p^{PIS+} - d_p^{PIS}(X)}{d_p^{PIS+} - d_p^{PIS-}} \tag{23}$$

$$\mu_p^{NIS} \leqslant \frac{d_p^{NIS}(X) - d_p^{NIS-}}{d_p^{NIS-} - d_p^{NIS+}} \tag{24}$$

$$1 \geqslant \mu_{\widetilde{R}_1(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_1 \tag{25}$$

$$2\left(\mu_p^{PIS} - \mu_p^{NIS} + 0.5\right) + M \times y \geqslant \mu_{\widetilde{R}_2(s,t)}, \quad \forall l_{st} = 1, \widetilde{R}(s,t) = \widetilde{R}_2 \tag{26}$$

$$-2\left(\mu_p^{PIS} - \mu_p^{NIS} - 0.5\right) - M \times y(1-z) \leqslant \mu_{\widetilde{R}_2(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_2 \tag{27}$$

$$2\left(\mu_p^{PIS} - \mu_p^{NIS} + 1\right) \geqslant \mu_{\widetilde{R}_3(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_3 \tag{28}$$

$$\left(\mu_p^{PIS} - \mu_p^{NIS} + 1\right) \geqslant \mu_{\widetilde{R}_4(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_4 \tag{29}$$

$$\frac{2}{3}\left(\mu_p^{PIS} - \mu_p^{NIS} + 1\right) \geqslant \mu_{\widetilde{R}_5(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_5 \tag{30}$$

$$\frac{(\mu_p^{PIS} - \mu_p^{NIS} + 1)}{2} \geqslant \mu_{\widetilde{R}_6(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_6 \tag{31}$$

$$\frac{2}{3}\left(\mu_p^{PIS} - \mu_p^{NIS} + 0.5\right) \geqslant \mu_{\widetilde{R}_7(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_7 \tag{32}$$

$$\left(\mu_p^{PIS} - \mu_p^{NIS}\right) \geqslant \mu_{\widetilde{R}_8(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_8 \tag{33}$$

$$2\left(\mu_p^{PIS} - \mu_p^{NIS} - 0.5\right) \geqslant \mu_{\widetilde{R}_9(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_9 \tag{34}$$

$$1 \geqslant \mu_{\widetilde{R}_{10}(s,t)}, \quad \forall l_{st} = 1, \ \widetilde{R}(s,t) = \widetilde{R}_{10} \tag{35}$$

$$0 \leqslant \mu_p^{PIS} \leqslant 1 \tag{36}$$

$$0 \leqslant \mu_p^{NIS} \leqslant 1 \tag{37}$$

$$0 \leqslant \mu_{\widetilde{R}(s,t)} \leqslant 1, \quad \forall l_{st} = 1, \quad s,t = 1,2, \quad s \neq t \tag{38}$$

$$l_{st} \in \{0,1\}, \quad s,t = 1,2, \quad s \neq t \tag{39}$$

$$0 \leqslant \lambda \leqslant 1 \tag{40}$$

$$w_p^{PIS} + w_p^{NIS} = 1 \tag{41}$$

$$X \in S \tag{42}$$

$$y \in \{0,1\} \tag{43}$$

We should note that s and t indices in model (22)–(43) refer to the PIS and the NIS in model (9)–(11), respectively. In addition:

- Objective function (22) simultaneously maximizes a convex combination of the weighted additive membership values of the fuzzy goals and the weighted sum of the uncertain DM's preferences on the membership values of the fuzzy goals.
- Inequalities (23) and (24) have been written for the first and the second fuzzy goals (i.e., $d_p^{PIS}(x)$ and $d_p^{NIS}(x)$).
- Only one of the constraints (25)–(35) is held at the same time. This reduces the complexity of model (22)–(43) as well as the complexity of the overall procedure and only one judgment about the DM's preferences on the membership values of these two goals is required.
- Relations (36)–(38) guarantee the lower- and the upper-bound of the membership values of the fuzzy goals and the fuzzy relations, respectively.
- Relation (39) holds the binary properties of the variable l_{st}, s, $t \in \{1,2\}$, $s \neq t$.
- Relation (40) controls the eligible values of parameter λ.
- Eq. (41) is used to control the sum of the weights as the parameters w_p^{PIS} and w_p^{NIS} help DM fine-tune the weight of the membership values of the fuzzy goals in the first segment of the objective function.
- Relation (42) represents the constraints in the original MODM problem (i.e., $g_j(X) \leqslant b$, $j = 1,2,\ldots,m$ in models (2)–(4).
- Relation (43) represents the binary properties of variable y.

The parametric nature of the proposed approach can help a DM generate arbitrary solutions with a desirable trade-off between the weighted additive membership values of the fuzzy goals and the weighted sum of the DM's preferences on the membership values of the fuzzy goals. As parameter λ in (22) increases, the objectives in the *TOPSIS-based bi-objective problem* are weighted more and the procedure tends to generate solutions which simultaneously satisfy the minimum distance from the PIS and the maximum distance from the NIS. In this case, the DM's preferences on the priority of the membership values of these two fuzzy objectives (i.e., $d_p^{PIS}(x)$ and $d_p^{NIS}(x)$) are weighted less and consequently the generated solutions are less likely of satisfying these preferences.

Next, a combined procedure (i.e., *Algorithm II*) is presented for solving the MODM problem. This procedure which is based on FGP and TOPSIS considers the uncertain DM's preferences on the priority of the membership values of the fuzzy goals which have been modeled using linguistic terms parameterized with linear fuzzy relation membership functions as introduced in Eqs. (12)–(21) and Fig. 2.

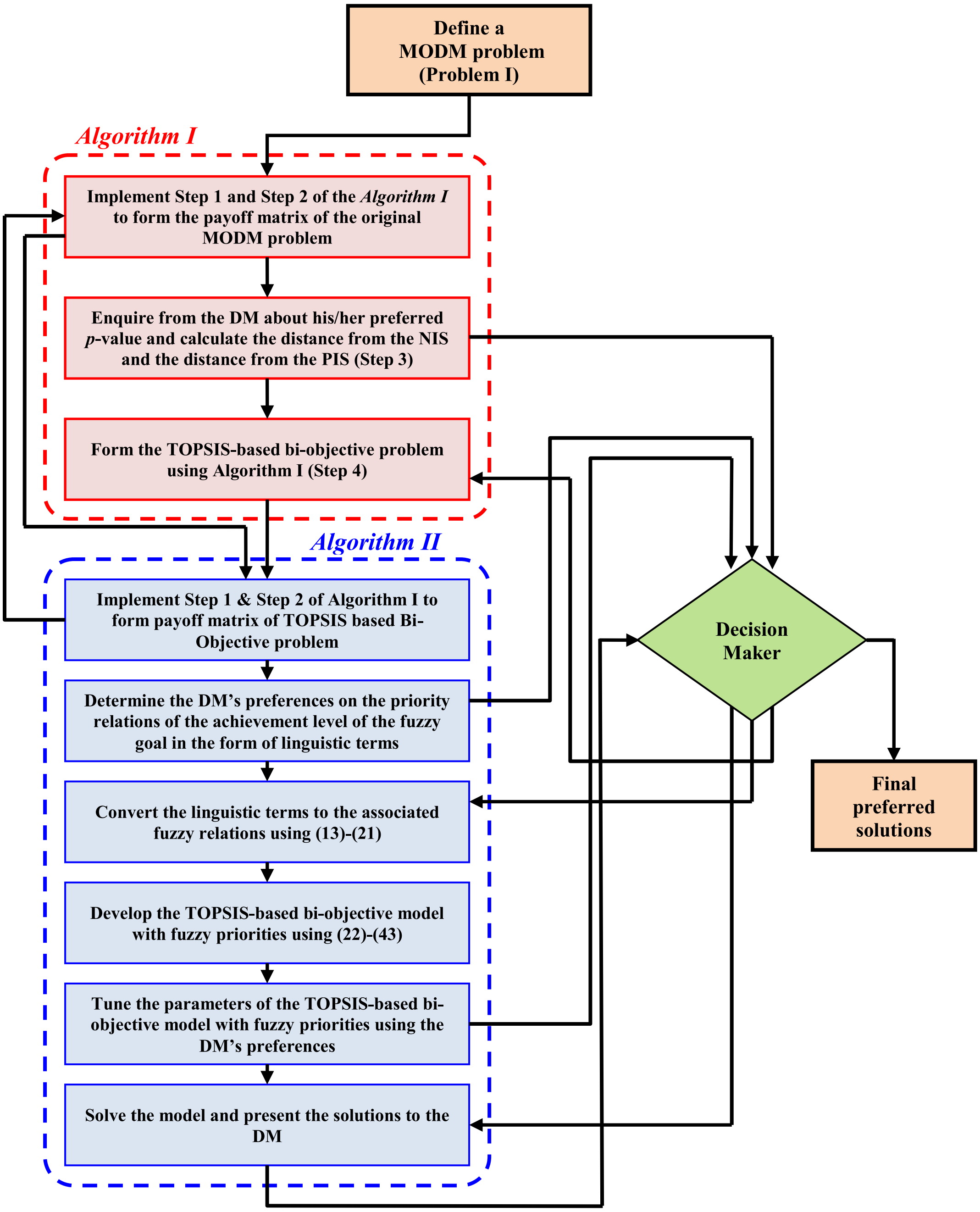


Fig. 3. Flowchart of the proposed framework.

3.4. Algorithm II

Step 1. Apply *Algorithm I* to the MODM *Problem (1)*.
Step 2. Form the pay-off table of the*TOPSIS-based bi-objective problem* (9)–(11).
Step 3. Ask the DM to determine his/her preferences on the priority of the membership values of the w_p^{PIS} and w_p^{NIS} objectives in *problem* (9)–(11).
Step 4. Form the *TOPSIS-based goal programming with fuzzy priorities* using model (22)–(43).
Step 5. Fine-tune the parameters in model (22)–(43), solve model (22)–(43), and present a set of solutions to the DM.

We should note that Fig. 3 presents the flowchart of the proposed framework as well as the interactions between the *Algorithms I*, the *Algorithm II*, and the DM.

4. Practical application of the proposed approach in the MOPS–MPPH problem

The selection of an optimum portfolio of projects has both practical and theoretical importance in project management. In this section we present the practical application for the proposed framework.

4.1. Brief review of the project selection literature using mathematical programming

Chan et al. [11] developed a goal seeking model for solving capital budgeting problems. Steuer and Na [61] provided a comprehensive and categorized bibliography of the multi-criteria decision making applications in capital budgeting, working capital management, and portfolio analysis. Badri et al. [8] developed a 0–1 GP model for information system project selection. Padberg and Wilczak [53] applied a mathematical programming model to obtain an optimal decision rule for project selection as a capital budgeting problem. Timothy and Kalu [65] developed a GP for capital budgeting under uncertainty. Xu et al. [73] presented a fuzzy 0–1 integer chance-constrained programming model to find the solution for multi-project multi-item investment problems. NSGA-II was used to solve an optimization model with a small modification to the constraint-handling rule. Gupta et al. [25] used fuzzy mathematical programming to develop a comprehensive set of models for asset portfolio optimization. Huang [32] studied the capital budgeting problem in fuzzy environments. Two types of models were proposed using the credibility to measure a confidence level. A fuzzy simulation-based genetic algorithm was applied to solve the problem. Huang [33] proposed an optimal portfolio selection model to solve the portfolio selection problem with random and fuzzy variables using neural networks. Liao and Ho [48] proposed a fuzzy binomial approach to evaluate projects under uncertainty. They developed a method to compute the mean value of a project's fuzzy net present value. Tiryaki and Ahlatcioglu [66] proposed a revised version of the fuzzy analytic hierarchy process to solve a classical portfolio selection problem. They applied their proposed method in choosing stocks on the Istanbul Stock Exchange. Zhang et al. [78] proposed a portfolio selection model based on the lower- and upper-possibilistic means and possibilistic variances. The MODM procedure (22)–(43) proposed in this study is applicable to a great many applications in multi-criteria decision making. Next, we demonstrate the applicability of the proposed MODM procedure to the MOPS–MPPH problem:

Let us consider an organization that is facing several investment opportunities in the form of projects with the following indices, parameters, and decision variables:

Indices:

j	Number of projects ($j = 1,2,\ldots,n$)
i	Type of human resources ($i = 1,2,\ldots,m$)
k	Type of machine ($k = 1,2,\ldots,s$)
o	Type of raw material ($o = 1,2,\ldots,z$)
t	Planning period ($t = 1,2,\ldots,T$)

Parameters:

H_{it}	Maximum available human resources Type i in Period t (man-hours)
h_{ij}	Requirement of human resources Type i in Project j (man-hours)
M_{kt}	Maximum available machine-hour Type k in Period t
m_{kj}	Requirement of machine-hour Type k in Project j
R_{ot}	Maximum available raw material Type o in Period t
r_{oj}	Requirement of raw material Type o in Period j
B_{jt}	Maximum available budget for project j in Period t
C_{it}	Hourly cost of human resources Type i in Period t
C_{kt}	Hourly cost of machine Type k in Period t
C_{ot}	Unit cost of material Type o in Period t
p_{jt}	Total net profit of Project j in Period t
I_{jt}	Rate of return for Project j in Period t

$MARR_t$	Minimum attractive rate of return in Period t
d_{jt}	Duration of project j in Period t

Decision variables:

$$x_{jt} = \begin{cases} 1 & \text{if Project } j \text{ is selected for investement in Period } t \\ 0 & \text{otherwise} \end{cases}$$

Model (44)–(57) attempts to select an optimum portfolio of investment projects in a fuzzy multi-objective space considering several constraints in the multi-period planning horizon. We should note that all parameters of the proposed mathematical model are assumed to be dynamically changeable through the planning horizon.

MOPS–MPPH problem objectives:

$$\text{Max } Z_1 = \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt} \times p_{jt} \tag{44}$$

$$\text{Min } Z_2 = \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt}\sum_{i=1}^{m} h_{ij}\cdot C_{it} + \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt}\sum_{k=1}^{s} m_{kj}\cdot C_{kt} + \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt}\sum_{o=1}^{z} r_{oj}\cdot C_{ot} \tag{45}$$

$$\text{Max } Z_3 = \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt} \times I_{jt} \tag{46}$$

$$\text{Min } Z_4 = \sum_{t=1}^{T}\left[\sum_{i=1}^{m}(H_{it} - \sum_{j=1}^{n} h_{ij}\cdot x_{jt}) + \sum_{k=1}^{s}(M_{kt} - \sum_{j=1}^{n} m_{kj}\cdot x_{jt}) + \sum_{o=1}^{z}(R_{ot} - \sum_{j=1}^{n} r_{oj}\cdot x_{jt})\right] \tag{47}$$

Constraints:

$$\sum_{t=1}^{T} x_{jt} \leqslant 1, \quad j = 1,2,\ldots,n \tag{48}$$

$$\sum_{t=1}^{T} (t + d_{jt})\cdot x_{jt} \leqslant T + 1, \quad j = 1,2,\ldots,n \tag{49}$$

$$\sum_{j=1}^{n} h_{ij}x_{jt} \leqslant H_{it}, \quad i = 1,2,\ldots,m, \quad t = 1,2,\ldots,T \tag{50}$$

$$\sum_{j=1}^{n} m_{kj}x_{jt} \leqslant M_{kt}, \quad k = 1,2,\ldots,s, \quad t = 1,2,\ldots,T \tag{51}$$

$$\sum_{j=1}^{n} r_{oj}x_{jt} \leqslant R_{ot}, \quad o = 1,2,\ldots,z, \quad t = 1,2,\ldots,T \tag{52}$$

$$\left(\sum_{i=1}^{m} h_{ij}\cdot C_{it} + \sum_{k=1}^{s} m_{kj}\cdot C_{kt} + \sum_{o=1}^{z} r_{oj}\cdot C_{ot}\right) \times x_{jt} \leqslant B_{jt}, \quad j = 1,2,\ldots,n, \quad t = 1,2,\ldots,T \tag{53}$$

$$\left(\sum_{i=1}^{m} h_{ij}\cdot C_{it} + \sum_{k=1}^{s} m_{kj}\cdot C_{kt} + \sum_{o=1}^{z} r_{oj}\cdot C_{ot}\right) \times x_{jt} < P_{jt}, \quad j = 1,2,\ldots,n, \quad t = 1,2,\ldots,T \tag{54}$$

$$\sum_{j=1}^{n} (x_{jt}\cdot (MARR_t - I_{jt})) \leqslant 0, \quad t = 1,2,\ldots,T \tag{55}$$

$$\sum_{j=1}^{n} x_{jt} \geqslant 0, \quad t = 1,2,\ldots,T \tag{56}$$

$$x_{jt} \in \{0, 1\}, \quad j = 1,2,\ldots,n, \quad t = 1,2,\ldots,T \tag{57}$$

We consider the following fuzzy objectives:

- Eq. (44) is used to maximize the net profit of the selected projects, where T is the upper-bound of the planning periods.
- Eq. (45) attempts to minimize the total cost of the selected projects, where s and z are reserved for the machine type and the raw material type, respectively.
- Eq. (46) is intended to maximize the total internal rate of return of the selected projects.
- The last objective function presented in Eq. (47) is used to minimize the total unused resources of the optimum portfolio, subject to the following constraints:

- Constraints (48), held for all projects, ensure that each selected project must be selected only one time throughout the planning horizon.
- Constraints (49), held for all projects, ensure that each selected project is completed in the planning horizon.
- Constraints (50), held for all human resources types in all projects and all planning periods, ensure that all human resources needs are met during the project selection process.
- Constraints (51), held for all machine-hour types in all projects and all planning periods, ensure that all machine-hour needs are met during the project selection process.
- Constraints (52), held for all raw material types in all projects and all planning periods, ensure that all raw material needs are met during the project selection process.
- Constraint (53), held for all projects and all planning periods, checks the budget availability in the project selection process.
- Constraint (54), held for all projects and all planning periods, ensures that the total cost of a selected project is less than its profit.
- Constraints (55) ensure that the internal rate of return of the selected projects is greater than or equal to the Minimum Attractive Rate of Return (MARR).
- Constraints (56) indicate that projects could be selected in each planning period.
- Constraints (57) refer to the zero-one orientation of the decision variables.

5. Experimental results

The performance of *Algorithm II* has been compared with an extended version of the FGP method proposed by Aköz and Petrovic [5] for random generated cases of MOPS–MPPH. Both procedures have been coded in Lingo 11.0 and executed on a Pentium 4 portable PC with Core 2 due CPU, 2 GHz, and Windows XP using 1 GB of RAM.

Table 2
Test problems.

Project number	Period number	H_{it}, M_{kt}, R_{ot}	h_{ij}, m_{kj}, r_{oj}	P_{jt}, B_{jt}	C_{it}, C_{kt}, C_{ot}	I_{jt}	$MARR_t$	d_{jt}
5	5	$U[0,10{,}000]$	$U[0,2]$	$U[1,100{,}000]$	$U[0,2]$	$U[1,10]$	$U[1,5]$	$U[1,3]$

Table 3
Budgets, profits, internal rates, durations, and MARR for the random case.

Project	Period 1	Period 2	Period 3	Period 4	Period 5
B_{jt}					
1	99668.00	29912.00	35258.00	15433.00	63050.00
2	72868.00	59889.00	97047.00	75272.00	20623.00
3	32079.00	96531.00	93877.00	12606.00	85533.00
4	10054.00	79387.00	28643.00	55232.00	74061.00
5	33993.00	71170.00	43455.00	65723.00	31798.00
P_{jt}					
1	8937.00	5646.00	5911.00	59855.00	30449.00
2	3096.00	51514.00	48050.00	54892.00	26798.00
3	81986.00	62308.00	73564.00	47882.00	89556.00
4	69612.00	27785.00	82869.00	26735.00	67944.00
5	76689.00	95638.00	33644.00	25946.00	26610.00
I_{jt}					
1	6.00	6.00	4.00	4.00	2.00
2	10.00	6.00	8.00	3.00	4.00
3	8.00	6.00	11.00	4.00	9.00
4	2.00	3.00	10.00	6.00	3.00
5	8.00	2.00	10.00	4.00	5.00
d_{jt}					
1	2.00	2.00	3.00	2.00	1.00
2	1.00	1.00	3.00	2.00	1.00
3	1.00	2.00	2.00	2.00	2.00
4	3.00	3.00	3.00	2.00	3.00
5	2.00	3.00	1.00	2.00	3.00
$MARR_t$	3	2	4	5	1

5.1. Generating the test problems

Table 2 shows the parameters for the generated cases. The human resources types, the machine-hour types, and the raw material types are represented by i, k, and o indices and are assumed to be equal to 4.

The generated random cases using the uniform distribution in Table 2 are summarized in Tables 3–6 provided in Appendix A.

5.2. Implementation of the proposed algorithm

Table 7 presents the payoff matrix of the single objective optimization in the original MODM problem. The range of the objective functions of the original MODM problem is presented in Table 7. These values are used to form the objective functions of the *TOPSIS-based bi-objective problem*and to set the target values of the goals in the method proposed by Aköz and Petrovic [5].

Table 8 shows the payoff matrix of the single objective optimization in the *TOPSIS-based bi-objective problem.* The range of the objective functions of the *TOPSIS-based bi-objective problem* is presented in Table 8. These values are used to set the target values of the goals in our proposed method. It is notable that the values of w_i, $i = 1, \ldots, 4$ and p are set to 0.25 and 1 in Tables 7

Table 4
Available resources for the random case.

Human	Period 1	Period 2	Period 3	Period 4	Period 5
H_{it}					
1	5919.00	7371.00	3761.00	8324.00	6228.00
2	2104.00	7253.00	9962.00	7437.00	1436.00
3	7769.00	4497.00	4433.00	1259.00	834.00
4	2282.00	5486.00	7777.00	4709.00	4134.00
Machine	Period 1	Period 2	Period 3	Period 4	Period 5
M_{kt}					
1	8401.00	4930.00	5343.00	9638.00	6598.00
2	4548.00	2567.00	8298.00	4915.00	758.00
3	4117.00	6528.00	2386.00	9824.00	5748.00
4	8402.00	1083.00	9120.00	1673.00	711.00
Material	Period 1	Period 2	Period 3	Period 4	Period 5
R_{ot}					
1	229.00	7323.00	7479.00	8336.00	7814.00
2	2413.00	9933.00	6460.00	1697.00	5967.00
3	8281.00	753.00	4277.00	4326.00	7098.00
4	1379.00	226.00	3053.00	2089.00	2715.00

Table 5
Required resources for the random case.

Project	Human 1	Human 2	Human 3	Human 4
h_{ij}				
1	0.00	1.00	1.00	1.00
2	1.00	2.00	1.00	2.00
3	0.00	0.00	1.00	1.00
4	0.00	1.00	0.00	1.00
5	1.00	1.00	1.00	0.00
	Machine 1	Machine 2	Machine 3	Machine 4
m_{kj}				
1	1.00	2.00	0.00	1.00
2	2.00	1.00	1.00	2.00
3	2.00	1.00	1.00	2.00
4	1.00	1.00	1.00	1.00
5	1.00	0.00	1.00	2.00
	Material 1	Material 2	Material 3	Material 4
r_{oj}				
1	1.00	0.00	0.00	1.00
2	1.00	1.00	1.00	1.00
3	0.00	1.00	0.00	0.00
4	1.00	1.00	2.00	1.00
5	2.00	2.00	2.00	1.00

Table 6
Unit cost of the resources for the random case.

Human	Period 1	Period 2	Period 3	Period 4	Period 5
C_{it}					
1	1.00	1.00	2.00	1.00	1.00
2	1.00	0.00	0.00	2.00	0.00
3	1.00	1.00	1.00	2.00	2.00
4	2.00	1.00	2.00	2.00	0.00
Machine	Period 1	Period 2	Period 3	Period 4	Period 5
C_{kt}					
1	0.00	1.00	1.00	1.00	0.00
2	1.00	1.00	2.00	0.00	2.00
3	1.00	1.00	1.00	1.00	1.00
4	0.00	1.00	2.00	1.00	2.00
Material	Period 1	Period 2	Period 3	Period 4	Period 5
C_{ot}					
1	0.00	0.00	0.00	2.00	1.00
2	0.00	1.00	1.00	2.00	0.00
3	0.00	0.00	0.00	2.00	2.00
4	2.00	0.00	1.00	1.00	2.00

Table 7
Payoff matrix of the single-objective optimization for Problem (1).

	Z_1	Z_2	Z_3	Z_4
Ideal calculations				
Max Z_1	327,879	60	15	300,352
Min Z_2	0	0	0	300,411
Max Z_3	161,366	83	21	300,352
Min Z_4	273,935	60	15	300,352
Anti-ideal Calculations				
Min Z_1	0	0	0	300,411
Max Z_2	215,310	83	21	300,352
Min Z_3	0	0	0	300,411
Max Z_4	0	0	0	300,411

Table 8
Payoff matrix of the TOPSIS-based bi-objective problem.

	d_p^{PIS}	d_p^{NIS}
Ideal calculations		
Min d_p^{PIS}	0.0100458	3.98995
Max d_p^{NIS}	0	3.98995
Anti-ideal calculations		
Max d_p^{PIS}	3	0
Min d_p^{NIS}	0	1

and 8, respectively. It can also be concluded from Table 8 that the range of the objective functions of the *TOPSIS-based bi-objective problem*has a narrow interval while this range in the original MODM problem had a wide interval in Table 7. In general, it is easier to compare a set of conflicting objectives in a narrow interval than a wide interval.

The DM's preferences on the priority of the membership values of the fuzzy goals in the original problem and the two goals in the *TOPSIS-based goal programming problem*are presented in Tables 9 and 10, respectively. The DMs independently selected an uncertain preference score for the priority of the membership values of the fuzzy goals. The values presented in Table 9 were used in an extended version of the FGP method proposed by Aköz and Petrovic [5] while the values in Table 10 were used in our proposed method.

Generally, the determination of the fuzzy relations is difficult, time consuming, and even impossible for MODM problems. In the proposed procedure, we reduce the multi-objective problem into a bi-objective problem to limit the number of comparisons made by the DMs to one. We should note that all the crisp weights were equally distributed in both procedures in order to establish a fair and comparable comparison setting.

Table 9
Fuzzy preference of the achievement level of the objectives in Problem (1).

	Z_1	Z_2	Z_3	Z_4
Z_1		R_3	R_3	R_3
Z_2	–		R_3	R_3
Z_3	–	–		R_3
Z_4	–	–	–	

Table 10
Fuzzy preference of the achievement level of the TOPSIS-based bi-objective problem.

	d_p^{PIS}	d_p^{NIS}
d_p^{PIS}		R_3
d_p^{NIS}	–	

Table 11
Results of the Aköz and Petrovic's procedure for the MOPS–MPPH case.

λ	$x_{jt} \mid x_{jt} \neq 0, \forall j, t$	μ_1	μ_2	μ_3	μ_4	$\sum_i w_i \mu_i$	$\sum_i \sum_j w_{ij} l_{ij} \mu_{\tilde{R}}(i,j)$	O.F.V.
0	$x_{13}, x_{34}, x_{44}, x_{52}$	0.7619,048	0.3809524	0.7619048	0.000000	0.4761905	0.9206349	0.9206349
0.1	$x_{13}, x_{34}, x_{44}, x_{52}$	0.8809524	0.3809524	0.7619048	0.000000	0.5059524	0.9206349	0.8791667
0.2	$x_{13}, x_{34}, x_{44}, x_{52}$	0.8809524	0.3809524	0.7619048	0.000000	0.5059524	0.9206349	0.8376984
0.3	$x_{13}, x_{34}, x_{44}, x_{52}$	0.8809524	0.3809524	0.7619048	0.000000	0.5059524	0.9206349	0.7962302
0.4	$x_{13}, x_{34}, x_{44}, x_{52}$	0.8809524	0.3809524	0.7619048	0.000000	0.5059524	0.9206349	0.7547619
0.5	$x_{13}, x_{34}, x_{44}, x_{52}$	0.9457831	0.4457831	0.7619048	0.1296615	0.5707831	0.8774144	0.7240988
0.6	x_{13}, x_{34}, x_{44}	1.000000	0.5421687	0.6666667	0.4176707	0.6566265	0.7496653	0.693842
0.7	x_{13}, x_{34}, x_{44}	1.000000	0.5421687	0.6666667	0.4176707	0.6566265	0.7496653	0.6845381
0.8	$x_{14}, x_{34}, x_{41}, x_{52}$	1.000000	0.5542169	0.5714286	0.7288136	0.7136147	0.5502689	0.6809455
0.9	$x_{14}, x_{34}, x_{41}, x_{52}$	1.000000	0.5542169	0.5714286	0.7288136	0.7136147	0.5502689	0.6972801
1	$x_{14}, x_{34}, x_{44}, x_{52}$	0.9457831	0.4457831	0.7619048	0.7288136	0.7205711	0.000000	0.7205711
Mean		0.925207	0.453554	0.709957	0.286495	0.593803	0.734587	0.76270615

Table 12
Results of the proposed algorithm for the MOPS–MPPH case.

λ	$x_{jt} \mid x_{jt} \neq 0, \forall j, t$	μ_p^{PIS}	μ_p^{NIS}	$\sum_i w_i \mu_i$	$\sum_i \sum_j w_{ij} l_{ij} \mu_{\tilde{R}}(i,j)$	O.F.V.
0	x_{24}, x_{34}	0.5000000	0.000000	0.2500000	1.000000	1
0.1	x_{52}	1.000000	0.5000000	0.7500000	1.000000	0.975
0.2	x_{52}	1.000000	0.5000000	0.7500000	1.000000	0.95
0.3	x_{52}	1.000000	0.5000000	0.7500000	1.000000	0.925
0.4	x_{52}	1.000000	0.5000000	0.7500000	1.000000	0.9
0.5	x_{52}	1.000000	0.5000000	0.7500000	1.000000	0.875
0.6	x_{52}	1.000000	1.000000	1.000000	0.6666667	0.8666667
0.7	x_{52}	1.000000	1.000000	1.000000	0.6666667	0.9
0.8	x_{52}	1.000000	1.000000	1.000000	0.6666667	0.9333333
0.9	x_{24}, x_{34}	1.000000	1.000000	1.000000	0.6666667	0.9666667
1	x_{24}, x_{34}	1.000000	1.000000	1.000000	0.6666667	1
Mean		0.954545	0.681818	0.818182	0.848485	0.9356061

The results from the extended version of the FGP method proposed by Aköz and Petrovic [5] and the results from our proposed method are summarized in Tables 11 and 12, respectively. The step-size of parameter λ is set equal to 0.1 in both procedures. All of the importance weights of the goals are assumed to be equally distributed for all segments of the objective functions in both methods to make a fair and comparable comparison.

It can be concluded from Tables 11 and 12 that the mean of the weighted additive membership values for the fuzzy goals in the proposed method is higher than the mean of the FGP method proposed by Aköz and Petrovic [5]. Moreover, it is obvious that the mean of the weighted additive membership values of the DMs' preferences on the uncertain priority of the fuzzy goals in the proposed method is relatively higher than the mean of the FGP method proposed by Aköz and Petrovic [5]. A narrow segment of the Pareto frontier which is both near the ideal solution and far from the anti-ideal solution is re-generated as the final solution since the proposed method uses TOPSIS to reduce the multi-objective space into a bi-objective space. Therefore, the multiplicity of the selected investment chances is reduced significantly. This results in the generation of more robust solutions under different circumstances. Moreover, the large amount of resources will be saved and could be made available for future use. It is notable that all of these occur where the satisfaction level of the fuzzy goals and the uncertain priority of the fuzzy goals are highly met in the proposed method.

5.3. Comparison indices

We have used a weighted sum membership value of the fuzzy goals, a weighted sum of the imprecise DMs' preferences on the priority of the membership values of the objectives, and the Closeness Coefficient (CC) to compare the performance of the two procedures. The CC is calculated as follows:

$$CC_{\lambda i} = \frac{d_p^{NIS_{\lambda i}}}{d_p^{NIS_{\lambda i}} + d_p^{PIS_{\lambda i}}}, \quad i \in \{1, 2\}, \quad \lambda \in [0, 1] \tag{58}$$

where $CC_{\lambda i}$ represents the CC of procedure i for parameter λ, $d_p^{NIS_{\lambda i}}$ represents the distance from the NIS for procedure i considering parameter λ and $d_p^{PIS_{\lambda i}}$ represents the distance from the PIS for procedure i considering parameter λ. It is obvious that $CC_{\lambda i}$ is between zero and one. The higher $CC_{\lambda i}$ values are associated with solutions which are simultaneously far from the NIS and close to the PIS. Table 13 shows the $CC_{\lambda i}$ values for the two procedures.

As is shown in this table, all the $CC_{\lambda i}$ values in our proposed method are relatively higher than the $CC_{\lambda i}$ values in the FGP method proposed by Aköz and Petrovic [5]. In other words, the solutions generated by our method are closer to the ideal solution and farther from the ant-ideal solution in comparison with the solutions generated by the FGP method proposed by Aköz and Petrovic [5]. Naturally, the case is the same for the mean row of the $CC_{\lambda i}$ values. We can conclude from Table 13 that the solutions generated by our method are superior to the solutions generated by the FGP method proposed by Aköz and Petrovic [5] with respect to the distance from the PIS, distance from the NIS, and the $CC_{\lambda i}$ values.

5.4. Analysis of variance (ANOVA) for the comparison indices

ANOVA experiments were conducted to compare the performance of the two procedures using the three aforementioned statistics (i.e., $\sum_i w_i \mu_i$, $\sum_i \sum_j w_{ij} l_{ij} \mu_{\tilde{R}}(i,j)$, and $CC_{\lambda i}$). In spite of the fact that Tables 11–13 present the apparent dominance of the proposed approach on these metrics, we tested the performance differences of the two procedures through ANOVA.

5.5. Interpreting the results of the ANOVA

An ANOVA test was conducted for 11 different samples of the aforementioned results which were a direct result of the step-size 0.1 for λ. The confidence levels of all experiments were set to 95%. The tests were accomplished using MINITAB 15.0 software.

Table 14 presents the results of the ANOVA for these metrics (i.e. $\sum_i w_i \mu_i$, $\sum_i \sum_j w_{ij} l_{ij} \mu_{\tilde{R}}(i,j)$, and $CC_{\lambda i}$). As is shown in this table, there is enough evidence to reject the hypothesis of equal means for the considered metrics. We can conclude that the achieved mean values of the selected measures (i.e. $\sum_i w_i \mu_i$, $\sum_i \sum_j w_{ij} l_{ij} \mu_{\tilde{R}}(i,j)$, and $CC_{\lambda i}$) in our proposed method are superior to the same measures in the FGP method proposed by Aköz and Petrovic [5].

Clearly, the p-values are less than the significant level (i.e., 0.05). The results of the ANOVA test and the results provided in Tables 11–13indicate that the performance of our proposed approach is significantly better than the performance of the FGP method proposed by Aköz and Petrovic [5].

Table 13
Closeness coefficients for both procedures.

λ	Aköz, and Petrovic [5]			Proposed framework (Algorithm II)		
	$d_p^{NIS_{\lambda 1}}$	$d_p^{PIS_{\lambda 1}}$	$CC_{\lambda 1}$	$d_p^{NIS_{\lambda 2}}$	$d_p^{PIS_{\lambda 2}}$	$CC_{\lambda 2}$
0	1.831907	2.103073	0.4655442	1.123279	1.160609	0.4918275
0.1	1.831907	2.103073	0.4655442	1.040843	1.050971	0.4975791
0.2	1.831907	2.103073	0.4655442	1.040843	1.050971	0.4975791
0.3	1.831907	2.103073	0.4655442	1.040843	1.050971	0.4975791
0.4	1.831907	2.103073	0.4655442	1.040843	1.050971	0.4975791
0.5	1.831907	2.103073	0.4655442	1.040843	1.050971	0.4975791
0.6	1.041694	1.052102	0.4975146	1.040843	1.050971	0.4975791
0.7	1.041694	1.052102	0.4975146	1.040843	1.050971	0.4975791
0.8	2.551417	3.060008	0.4546825	1.040843	1.050971	0.4975791
0.9	2.551417	3.060008	0.4546825	1.123279	1.160609	0.4918275
1	2.244240	2.651468	0.4584097	1.123279	1.160609	0.4918275
Mean	1.856537	2.135830	0.4687340	1.063326	1.080872	0.4960100

Table 14
Analysis of variance.

Source	Degree of freedom	Sum of square	Mean square	F	p-Value
I. First metric: $\sum_i w_i \mu_i$					
Algorithm II	2	0.090291	0.045146	47.43	0.000
Error	8	0.007615	0.000952		
Total	10	0.097907			
S = 0.03085 R-Sq = 92.22% R-Sq (adj.) = 90.28%					
II. Second metric: $\sum_i \sum_j w_{ij} l_{ij} \mu_{\tilde{R}}(i,j)$					
Algorithm II	1	0.4222	0.4222	10.02	0.011
Error	9	0.3793	0.0421		
Total	10	0.8015			
S = 0.2053 R-Sq = 52.68% R-Sq (adj.) = 47.42%					
III. Third metric: $CC_{\lambda i}$					
Algorithm II	1	0.000348	0.000348	3.68	0.048
Error	9	0.001871	0.000208		
Total	10	0.002219			
S = 0.01442 R-Sq = 15.69% R-Sq (adj.) = 6.32%					

6. Conclusion remarks

We proposed a FGP approach based on TOPSIS for MODM by considering uncertain DM's preferences on the priority of the membership values of the fuzzy goals. The proposed approach has several attractive features. The first feature is the substantial reduction of the objective function space. The proposed procedure reduces a multi-objective decision making problem to an efficient bi-objective problem. As a result, the computational efforts and complexities are reduced significantly and high-quality solutions that are simultaneously close to the PIS and far from the NIS are generated. The resulting bi-objective problem is solved using a new FGP. The second feature is the consideration of uncertain DMs' preferences on the priority of the membership values of the fuzzy goals in the form of fuzzy relations. The membership value of each fuzzy goal is modeled using fuzzy sets. Moreover, the fuzzy DM's preferences on the priority of the membership values of the fuzzy goals are effectively extended through a structure which is comprised of linguistic terms, fuzzy relations, and fuzzy membership functions. The third feature is the efficient consideration of the DMs' preferences. DMs often have a difficult time presenting their preferences on the priority of the fuzzy membership values which are uncertain themselves with crisp numbers. We model the uncertain preferences between the priorities of the membership values of the fuzzy goals using linguistic terms parameterized with linear fuzzy relations. The membership value of a given fuzzy goal is a fuzzy set with a membership value between zero and one. A fuzzy relation has been organized between the membership values of the two fuzzy goals with uncertainties. Ten different linguistic terms were introduced to model the uncertain preferences of the DMs on the priority of the membership values of the two fuzzy goals. Each linguistic term was associated with a linear type of fuzzy set which could have different values with predefined membership values. Each fuzzy relation represented the membership values of the uncertain preferences of the DMs on the priority of the membership values of the fuzzy goals, which are shown with the difference between the membership values of the competing fuzzy goals i and j, $\mu_i - \mu_j$ through a continuous fuzzy set. In each fuzzy relation set, the more conformity of $\mu_i - \mu_j$ with its associated linguistic term is, the higher the membership value of the relation is. This type of uncertainty modeling is a real application of type II fuzzy sets in which a fuzzy relation has been organized between membership values of the fuzzy goals which are fuzzy sets themselves.

The optimization model maximizes a parametric combination of the weighted sum membership values of the fuzzy goals and the weighted sum of the fuzzy priority relations of the membership values of the fuzzy goals. The aforementioned features of the proposed approach make it robust and well-posed for modeling real-life problems through FGP. Moreover the proposed procedure interacts with the DM to determine the fuzzy relation type and the fine-tuning parameter λ which sets the two main arguments of the objective functions in the proposed approach (i.e., the weighted sum of the membership values of the fuzzy goals and the weighted sum of the DM's preferences on the priority of the membership values of the fuzzy goals).

A multi-objective fuzzy binary optimization model for solving the MOPS–MPPH was developed to present the efficiency of the proposed procedure. In order to show the robustness of the proposed approach in solving complicated real-life problems, the MOPS–MPPH was used and compared with a FGP method proposed by Aköz and Petrovic [5]. The MOPS–MPPH was equipped with 4 conflicting fuzzy objective functions (i.e., benefit, cost, internal rate of return, and resource utilization). The problem involved seeking solutions for the four aforementioned fuzzy objective functions by considering several sets of constraints in a multi-period planning horizon. Three metrics were calculated in an experiment to compare the performance of the two models. *The proposed approach* dominated the extended version of the FGP method proposed by Aköz and Petrovic [5] on random benchmark cases. The structural and practical dominance of the proposed procedure are described next.

The relative importance of the fuzzy goals had not been considered in [5]. Our proposed approach reduces the multiple dimensions of the objective function space based on TOPSIS while considering the relative importance of the fuzzy goals (i.e.,

w_p^{PIS} and w_p^{NIS}). We also considered the relative importance of the DM's preferences on the membership values of the fuzzy goals i and j (i.e., w_{ij}). The number of independent DM's judgments on the priority of the membership values of the fuzzy goals was reduced to one in our proposed procedure while this is proportional to the number of objective functions in [5]. We developed a framework and a hierarchical structure of the DMs' preferences on the membership values of the fuzzy goals including linguistic terms and sub-terms, fuzzy relations, and the associated membership functions.

The proposed approach can efficiently generate solutions which are close to the PIS and far from the NIS, simultaneously. This was demonstrated through the ANOVA experiments and the $CC_{\lambda i}$ metric. The ANOVA experiments showed that the weighted sum of the membership values of the fuzzy goals in the proposed procedure are higher than in the same metric in [5].

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Appendix A. Generated random case for the MOPS–MPPH problem

See Tables 3–6.

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