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A logit-based model for measuring the effects of transportation infrastructure on land value

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ABSTRACT

Mutual interactions between transportation and land use have long been debated. Despite progress made in computational technology, the study of these interactions is not adequately developed. The most important aspect of such interactions is given by the changes in land values due to changes in transportation infrastructures. We consider the behavioural features of these interactions along with the constraints on the land and/or zoning restrictions and propose a reliable model for the first time to predict land value changes with respect to changes in transportation facilities and accessibility. The proposed model is a logit-based mathematical programming methodology where the relative price of land is predicted with respect to transportation accessibility, neighbourhood amenities, location premium, availability of land, and zoning regulations. A real-world case study is used to exhibit the applicability of the proposed methodology and demonstrate the efficacy of the algorithms and procedures.

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1. Introduction

The strong link existing between land use and transportation and their geographical as well as spatial connections have attracted many parties: policy-makers, transportation planners, urban designers, zoning authorities, and real estate investors. One of the most interesting aspects of this link is given by the changes in land value in response to changes in transportation provisions. Transportation projects aim to enhance accessibility of land uses. Improving the accessibility increases the attractiveness of land use, and hence, increases value of lands/properties. Such added-value is the result of public funding (not landlords') effort and, as such, it can be returned to the public sector to finance more projects. One way to collect this windfall revenue is to levy an additional tax on affected lands or properties; this is called 'value capture' (VC) (Iacono and Levinson 2011; Rolon 2008). Alternatively, some advocate tapping directly into the accessibility by constructing high

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demanding land uses in the premium locations. This corresponds to the backbone concept of 'transit-oriented development' (TOD), according to which the demanding developments such as commercial land uses are constructed in the vicinity of transit hubs (like metro stations).

VC and TOD are two widespread concepts for which the precise assessment of potential changes in land value is essential. There have been many attempts to create a methodology for this assessment, but a unified consensus has yet to be developed. The most common methods used to predict property values are based on hedonic regression models, in which the values of properties are broken down into components such as location, construction material, structural features (i.e. fireplaces, basements), heating, ventilation and air conditioning systems, amenities, mortgage interest rates, zoning regulations, and accessibility (California-Department-of-Transportation 2013; Carey and Semmens 2003; Herath and Maier 2010; Iacono and Levinson 2011; Rolon 2008). In such methods, the most significant deficiency is represented by the loose treatment of availability constraints and regulations imposed by the market. It is evident that in a supply–demand market like real estate, the value of a product is strongly driven by its availability. Furthermore, the behaviour of customers when choosing appropriate property has been largely ignored (Iacono, Levinson, and El-Geneidy 2008). It is important to capture customers' preferences, how they choose a property, and most importantly, their competitive behaviour.

In this study, we investigate changes in property values in response to changes in accessibility due to changes in transportation infrastructure. The aforementioned methodological deficiencies are addressed. First, customers' behaviour is captured by adopting a logit model for land use choice. Logit land use choice models comprise factors affecting utility of choices such as price, amenities, etc., as well as factors pertaining to the transportation infrastructure such as accessibility. Second, we develop a mathematical programming model embedded with the Logit model to ensure that market competition is accounted for. In this model, a variety of supply-side constraints are explicitly introduced. Marginal (or shadow) prices of these constraints are utilized to screen changes in land values due to changes in transportation facilities. The proposed methodology has been widely applied to analyse situations dealing with the concept of choice versus limited choice options, such as parking modelling and pricing (Bagloee Asadi and Asadi 2013; Bagloee Asadi, Asadi, and Richardson 2012); facility placement in supply chain management (Bagloee Asadi et al. 2015); credit rationing in loan market (Bagloee Asadi, Asadi, and Mohebbi 2014); and budget allocation (Bagloee Asadi and Reddick 2011).

2. Literature review

We will first review different modelling approaches existing in the literature. Afterwards, we will review several aspects of value changes in the market and their financial implications on transportation.

2.1. Modelling approaches

Since the pioneer work of Lowry (1964), a significant effort has been made to integrate transportation and land use models and to understand their co-evolution (Bina, Warburg, and Kockelman 2006; Iacono and Levinson 2011; Iacono, Levinson, and El-Geneidy 2008; Sivakumar 2007; Yim et al. 2011). Integrated models have been even mandated in some jurisdictions (Krishnamurthy and Kockelman 2003; May et al. 2005; Vadali, Aldrete, and Bujanda 2009; Waddell et al. 2007). Nevertheless, the literature has yet to provide a common view on the matter.

A wide range of techniques has been employed to screen real estate values, including empirical studies, trend-based extrapolation, modelling, growth assessment, statistical regression methods, Markov chains, and activity-based models (Doherty 2004; Vadali, Aldrete, and Bujanda 2009). Most of the efforts have relied on large datasets of previous studies, which is a prohibitive factor in view of real applications (Levinson and Chen 2005; Sivakumar 2007).

Hedonic regression models have been the most used ones. These models break down property characteristics into component attributes, and then explain how consumers value the different attributes (Du and Mulley 2006; Economics Research Associates 2006; Vadali and Sohn 2001). Hedonic models have gained popularity due to their simplicity and intuitive appeal (Herath and Maier 2010; Iacono, Levinson, and El-Geneidy 2008; Kawamura and Mahajan 2005; Medda 2012). Netusil (2013) reviewed recent advances in hedonic models. Some integrated models and commercial land use planning applications entail a module to simulate the real estate market (Anas and Liu 2007; Iacono and Levinson 2011; Miller 2003; Salvini and Miller 2005; Waddell 2000; Waddell et al. 2007) in which the values are determined internally by hedonic methods or bid-rent methods. These models require many interacting sub-modules for which calibration is a daunting task (Iacono, Levinson, and El-Geneidy 2008; Ma and Lo 2012). The bid-rent model is an auctioning practice that leads to market equilibrium, which is a challenging and controversial assumption (Bowman 2006; de Palma, Picard, and Waddell 2007; Iacono, Levinson, and El-Geneidy 2008; Ma and Lo 2012; Martínez and Henríquez 2007; Miller 2003). In both hedonic and bid-rent models, what is missing the most is taking into account customers' behaviour when choosing land use (de Palma, Picard, and Waddell 2007; Iacono, Levinson, and El-Geneidy 2008) for which, alternatively, random utility models represented by logit models are suggested (de Palma, Picard, and Waddell 2007; Iacono, Levinson, and El-Geneidy 2008; Miller 2003). Logit models have been widely used in land use choice modelling and commercial applications (de Palma, Picard, and Waddell 2007; Iacono, Levinson, and El-Geneidy 2008; Wang, Kockelman, and Wang 2011; Zhao and Peng 2012).

de Palma, Picard, and Waddell (2007) proposed a logit model with capacity constraints for analysing the housing market. The included housing price model is a hedonic regression model. Customers' choices are subject to the availability of housing units. However, the proposed methodology relies on two assumptions that limit its application: first, the demand for each and every unit in the market is assumed to be known; and second, the markets for which the methodology is developed are basically assumed to be homogeneous (solutions for the heterogeneous cases were derived, but their reliability has yet to be investigated).

2.2. Land value and its financial implications

A vast pool of studies is available with empirical findings suggesting that transportation projects, especially urban rail, have a significant impact on property values (Billings

2011; Kawamura and Mahajan 2005; Levinson and Chen 2005). A considerable share of the literature is devoted to transit infrastructure, most notably rail projects (Rolon 2008). Most studies show a positive impact on real estate value when location varies relative to proximity to transit centres (California-Department-of-Transportation 2013; Doherty 2004; Economics Research Associates 2006; Mohammad et al. 2013; Pagliara and Papa 2011). Furthermore, among these properties, commercial land uses benefit more than residential land uses (Mohammad et al. 2013; Rolon 2008; Weinberger 2001). Such characteristics provide great opportunity to mandate the TOD concept (Doherty 2004; Rolon 2008). Similar observations were reported for road projects (California-Department-of-Transportation 2013; Carey and Semmens 2003; Levinson and Istrate 2011). Mohammad et al. (2013) provides a comprehensive discussion and review of the transit and real estate market literature.

Traditionally, transportation projects rely on fuel taxes or general revenues for both capital and maintenance expenditures (Batt 2001). An alternative is to tap into the value added to real estate that accrues from improvements in transportation infrastructure through VC (Doherty 2004; Du and Mulley 2006; Levinson and Istrate 2011; Medda 2012; Rolon 2008; Vadali, Aldrete, and Bujanda 2009). Medda (2012) reviewed the literature and applications pertaining to VC. He found that predicting changes in land value can suggest changes in public revenue from property taxes, since there is a strong correlation between property values and taxes (Lang and Jian 2004). Equally important to the success of a VC tax is the accurate assessment of real estate values (Doherty 2004; Junge and Levinson 2012a; Rolon 2008; Vadali, Aldrete, and Bujanda 2009).

At the same time, land uses that increase traffic or that use more of an existing transportation infrastructure are expected to pay proportionally more maintenance expenses through transportation utility fees. Some researchers suggested such a fee in lieu of a tax since people prefer to pay a fee proportionate to their consumption (Junge and Levinson 2012b). However, legal issues and the difficulty of measuring transportation facility usage have made the transportation utility fee unsuccessful in practice (Carlson et al. 2007; Herath and Maier 2010).

3. Modelling the land use problem

Let G_{pq} be the demand for land use type $q \in Q$ made by customer $p \in P$ who have to find a location zone denoted by $k \in K$. Here, G_{pq} is an exogenously determined land use demand.

We consider heterogeneous types of customers. An element $p \in P$ may represent an individual customer or a group of customers; different classes or types of customers will be represented as elements of a set of customer groups: $P = \{p_1, p_2, \ldots\}$. In the case of homogeneous customers (which represents an unrealistic or simplified situation), the index p can be dropped and G_q used. Likewise, $q \in Q$ can represent various types of land use such as commercial, retail, residential, and even transaction contracts (lease or sale): $Q = \{q_1, q_2, \ldots\}$. Therefore, both ends of the land use demands are considered heterogeneously: G_{pq} is the demand for $p \in P$ customers and $q \in Q$ land use types.

In the end, a customer selects a best possible unit located in a place or zone ($k \in K$) subject to availability, characteristics of the choice, and his/her preferences. Thus, the element $k \in K$ represents locations, which could be a community, a zone, a block, or even an individual unit, depending on the extent of (dis)aggregations. For easier reference, we call it 'zone' ($k \in K$), which comprises one or many units with various land use types $q \in Q$.

Customers choose the locations that provide maximum utility. Utility is the core concept of discrete choice models, including logit models (Bagloee Asadi, Asadi, and Mohebbi 2014). The factors influencing customers' choice may include lease/sale asking price, property tax rate, housing (municipality) fees, security, closeness to amenities, characteristics of neighbours, age, type, and finish of buildings, and accessibility (Bina, Warburg, and Kockelman 2006; Du and Mulley 2006). Accessibility stands for the impact of transportation infrastructure as a significant externality factor in customer choice. The utility associated with a certain choice is usually defined as a linear function of the factors and called a utility function. The factors are associated with coefficients indicating the relative importance of each factor in the choosing process. The utility derived from choosing zone $k \in K$ is perceived by each customer in a different way. Depending on their socio-economical characteristics and their individual preferences, customers may assign different levels of utility to the same zone. Therefore, we will denote by u_{pkq} the utility that the customer p believes he/she will derive from choosing a unit of land use type q in zone k.

Regardless of any restriction in availability and zoning regulations, the logit land choice (LLC) model may be formulated as follows:

$$g_{pkq} = G_{pq} \times \frac{\mathrm{e}^{u_{pkq}}}{\sum_{k \in K} \mathrm{e}^{u_{pkq}}}, \tag{1}$$

where g_{pkq} is the number of customers in class p out of the total land use demand G_{pq} who would likely choose a unit of land use type q located in zone k.

In each zone k, there is a limited number of units. We call this limit the capacity of the zone and denote it by C_k. The zone capacity consists of units rationed for different land use types. Since only zoning authorities are entitled to enforce such land use rationing, we also need to consider 'zoning' (or 'rationing') restrictions. Therefore, constraints for capacity and land use rationing are introduced as follows:

Land use capacity constraints (LCCs):

$$\sum_{p \in P, q \in Q} g_{pkq} \leq C_k, \quad k \in K. \tag{2}$$

Land rationing constraints (LRCs):

$$\sum_{p \in P} g_{pkq} \leq F_{kq}, \quad q \in Q, \; k \in K, \tag{3}$$

with Equation (2) we make sure that total unit acquisition (or transaction, or occupation) $\left(\sum_{p \in P, q \in Q} g_{pkq}\right)$ does not exceed the total C_k of units available in the market in the zone k. Zoning restrictions are considered in Equation (3): the maximum number of units in zone k rationed to land use type q is denoted by F_{kq}. For instance, zoning authorities may consider the designation of 100 units for residential units, 20 units for retail, and 10 units for commercial usage at a particular zone. Therefore, the land use modelling is transferred to the LLC problem subject to LCCs and LRCs (i.e. Equation (1) subjects to Equations (2) and (3)). The inputs of the problem are summarized as follows:

G_{pq}: Land use type q demand by customer group p, or how many customers from class p are interested in land use type q;

u_{pkq}: Utility of choosing a unit of land use type q at zone k perceived by the customers in class p;

C_k: Capacity or maximum available units at zone k; and

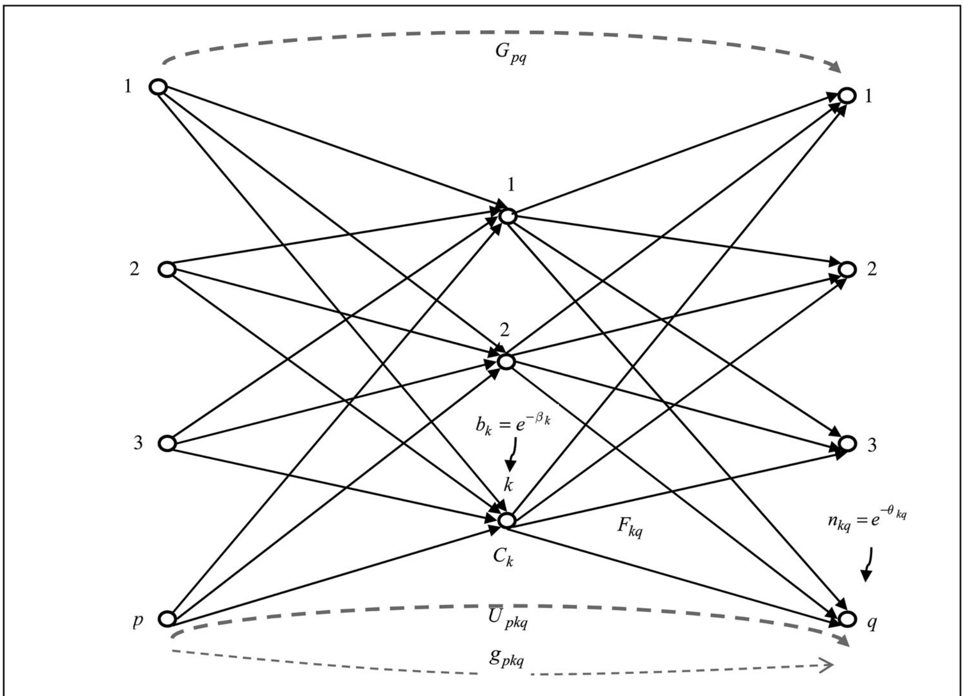
F_{kq}: Land use zoning, or maximum units of zone k rationed to land use type q.

Figure 1 presents a graphical illustration of the model structure. The primary output is g_{pkq}, that is, how many customers in class p (out of G_{pq}) looking for land use type q have chosen units in zone k.

4. Proposed methodology

4.1. Logit-based mathematical programming for land use modelling

Spiess (1996) was the first scholar to mathematically solve the problem formalized as 'Equation (1) *s.t.* Equation (2)' without the zoning restriction (Equation (3)). By applying



Inputs	Outputs
***Gpq*; land use demand: number of customers of class p interested in land use type q.	***g_{pkq}*; transaction volume: number of customers of class p interested in land use type q who chose units in zone k
***U_{pkq}=exp(-u_{pkq})*; utility in exponential form perceived by customers of class p when choosing a unit of type q in zone k.	***β_k*; shadow price corresponding to supplied capacity at zone k
***C_k*; capacity of zone k : number of units available in zone k	***θ_{kq}*; shadow price corresponding to rationed capacity of zone k for land use type q
***F_{kq}*; land use rationing: maximum units in zone k dedicated to land use type q	

Figure 1. Structure, inputs, and outputs of land use modelling.

the Kuhn–Tucker optimality conditions, Spiess (1996) first proved that the problem represented by Equation (1) in the absence of any constraints is equivalent to the following minimization problem:

$$\text{Min} \sum_{\substack{p\in P,k\in K\\ q\in Q}} g_{pkq}(\log g_{pkq} - 1 + u_{pkq}), \tag{4}$$

s.t.

$$\sum_{k\in K} g_{pkq} = G_{pq}, \quad p \in P, \ q \in Q. \tag{5}$$

Spiess (1996) then added the constraints given by Equation (2) to the above problem. Considering explicit capacity constraints (LCCs), the LLC problem becomes:

$$\text{Min} \sum_{\substack{p\in P,k\in K\\ q\in Q}} g_{pkq}(\log g_{pkq} - 1 + u_{pkq}), \tag{6}$$

s.t.

$$\sum_{k\in K} g_{pkq} = G_{pq}, \quad p \in P, \ q \in Q, \tag{7}$$

$$\sum_{p\in P,\ q\in Q} g_{pkq} \le C_k, \quad k \in K. \tag{8}$$

We call the above problem (Equations (6)–(8)) the Spiess Problem. Equation (7) ensures that the outcome (g_{pkq}) meets the demand (G_{pq}). Equation (8) provides the LCCs.

It is necessary to ensure the feasibility of solutions to the Spiess Problem by providing enough supply to serve the entire demand, as follows:

$$\sum_{p\in P,q\in Q} G_{pq} \le \sum_{k\in K} C_k. \tag{9}$$

Clearly, in case of a shortage in the market, we can assume a dummy zone with infinite capacity and minimum utility. Minimum utility ensures that the dummy zone will not receive any demand unless real supply is utilized. Since Equation (9) can be treated beforehand, it was excluded from the formulation of the Spiess Problem; reducing the number of constraints makes the problem more tractable.

The Spiess Problem can be extended to accommodate land use rationing (zoning) constraints (LRCs) as follows, which we refer to as the land modelling (LM) problem:

$$\text{Min} \sum_{\substack{p\in P,k\in K\\ q\in Q}} g_{pkq}(\log g_{pkq} - 1 + u_{pkq}), \tag{10}$$

s.t.

$$\sum_{k\in K} g_{pkq} = G_{pq}, \quad p \in P, \ q \in Q, \tag{11}$$

$$\sum_{p \in P, q \in Q} g_{pkq} \leq C_k, \quad k \in K, \tag{12}$$

$$\sum_{p \in P} g_{pkq} \leq F_{kq}, \quad q \in Q, \ k \in K. \tag{13}$$

The presence of the LRCs (Equation (13)) in the above problem leads to a feasibility discussion similar to the Spiess Problem. Ensuring the feasibility of the solutions to the LM problem requires providing adequate rationing rates (F_{kq}) with respect to the available capacity (C_k), and meeting the entire demand (G_{pq}) as follows:

$$C_k \leq \sum_{q \in Q} F_{kq}, \quad k \in K, \tag{14}$$

$$\sum_{p \in P} G_{pq} \leq \sum_{k \in K} F_{kq}, \quad q \in Q. \tag{15}$$

At the same time, we must ensure feasibility of the solution beforehand by providing a dummy zone k' with infinite capacity ($C_{k'} = \infty$), minimum utility ($u_{pk'q} < \min\{u_{pkq} | p \in P, \ q \in Q, \ k \in K - \{k'\}\}$), and no rationing restriction ($F_{k'q} = \infty | q \in Q$). Furthermore, in order to avoid problems of degeneracy we need to assume that $C_k > 0$, for every $k \in K$.

4.2. Solution algorithm

We now focus our attention on establishing a solution algorithm for the LM problem. All the constraints in the LM problem are linear. Hence, we can manage to achieve a more tractable problem by deriving the dual format of the problem. To do so, we introduce the dual variables α_{pq} for the constraints of Equation (11) and $\beta_k \geq 0$ and $\theta_{kq} \geq 0$ for the constraints of Equations (12) and (13), respectively. The Kuhn–Tucker objective operator is then defined as follows:

$$\begin{aligned} L = & \sum_{\substack{p \in P, k \in K \\ q \in Q}} g_{pkq}(\log g_{pkq} - 1 + u_{pkq}) + \alpha_{pq}\left(\sum_{k \in K} g_{pkq} - G_{pq}\right) \\ & + \beta_k\left(\sum_{p \in P, \ q \in Q} g_{pkq} - C_k\right) + \theta_{kq}\left(\sum_{p \in P} g_{pkq} - F_{kq}\right). \end{aligned} \tag{16}$$

Hence, first-order Kuhn–Tucker optimality conditions may be written as:

$$\nabla L_{g_{pkq}} = 0 \Rightarrow \log g_{pkq} - 1 + u_{pkq} + g_{pkq} \cdot \frac{1}{g_{pkq}} + \alpha_{pq} + \beta_k + \theta_{kq} = 0. \tag{17}$$

Subjecting the transaction flow (g_{pkq}) derived from Equation (17) to the constraints of the LM problem (Equations (11)–(13)) results in a new problem, as follows:

$$g_{pkq} = \mathrm{e}^{-u_{pkq} - \alpha_{pq} - \beta_k - \theta_{kq}}, \tag{18}$$

s.t.

Equations (11)–(13),

Now we can establish the dual format of the LM problem as follows:

$$D = \underset{\alpha,\beta,\theta}{\mathrm{Min}} \sum_{\substack{p\in P,k\in K\\ q\in Q}} e^{-u_{pkq}-\alpha_{pq}-\beta_k-\theta_{kq}} + \sum_{p\in P,q\in Q} \alpha_{pq}G_{pq} + \sum_{k\in K}\beta_k C_k + \sum_{k\in K}\theta_{kq}F_{kq}, \quad (19)$$

s.t.

$$\begin{aligned} \beta_k &\geq 0, \quad k\in K,\\ \theta_{kq} &\geq 0, \quad k\in K,\ q\in Q. \end{aligned} \quad (20)$$

Again, computing the first-order Kuhn–Tucker optimality conditions for the dual problem, we have:

$$\nabla D_{\alpha_{pq}} = 0 \Rightarrow \sum_{k\in K} \mathrm{e}^{-u_{pkq}-\alpha_{pq}-\beta_k-\theta_{kq}} = G_{pq}, \quad p\in P,\ q\in Q, \quad (21)$$

$$\nabla D_{\beta_k} = 0 \Rightarrow \sum_{p\in P,\ q\in Q} \mathrm{e}^{-u_{pkq}-\alpha_{pq}-\beta_k-\theta_{kq}} \leq C_k, \quad k\in K, \quad (22)$$

$$\nabla D_{\theta_{kq}} = 0 \Rightarrow \sum_{p\in P} \mathrm{e}^{-u_{pkq}-\alpha_{pq}-\beta_k-\theta_{kq}} \leq F_{kq}, \quad q\in Q,\ k\in K. \quad (23)$$

Solving the above equations yields optimal values of the dual variables.

Spiess (1996) developed a solution algorithm using the successive coordinate descent (SCD) method for his own dual problem, since the dual problem was free of any explicit constraints. The same argument applies to our dual problem Equation (19). Therefore, we developed a solution algorithm based on the SCD method discussed below.

To reduce computational complexity, we eliminate the exponential terms in the optimality conditions (Equations (21)–(23)) through some simple substitutions:

$$a_{pq} = \mathrm{e}^{-\alpha_{pq}}, \quad b_k = \mathrm{e}^{-\beta_k},\ n_{kq} = \mathrm{e}^{-\theta_{kq}} \quad \text{and} \quad U_{pkq} = \mathrm{e}^{-u_{pkq}}. \quad (24)$$

Hence the non-negativity conditions of the dual variables become:

$$a_{pq} > 0, \quad 0 < b_k \leq 1, \quad 0 < n_{kq} \leq 1. \quad (25)$$

Now the optimality condition of the dual problem can be rewritten as:

$$\sum_{k\in K} U_{pkq}\cdot a_{pq}\cdot b_k\cdot n_{kq} = G_{pq}, \quad p\in P,\ q\in Q, \quad (26)$$

$$\sum_{p\in P,q\in Q} U_{pkq}\cdot a_{pq}\cdot b_k\cdot n_{kq} \leq C_k, \quad k\in K, \quad (27)$$

$$\sum_{p\in P} U_{pkq}\cdot a_{pq}\cdot b_k\cdot n_{kq} \leq F_{kq}, \quad q\in Q,\ k\in K. \quad (28)$$

The SCD is an iterative process. At the end of each iteration, the values of the dual variables (a_{pq}, b_k, n_{kq}) are updated for the next iteration. The superindex i will denote the next iteration values of the dual variables (a^i_{pq}, b^i_k, n^i_{kq}). The maximum number of iterations will be denoted by $i_{\max}$.

The SCD algorithm of the dual problem Equation (19) can be outlined as follows:

Solution Algorithm Outline

Step 0. Initialization and preparation

Set $i = 1$ and $a^0_{pq} = b^0_k = n^0_{kq} = 1$, for every $p \in P$, $k \in K$, $q \in Q$.

Step 1. Computation

For every $p \in P$, $k \in K$ and $q \in Q$, compute:

$$a^i_{pq} = \frac{G_{pq}}{\sum_{k\in K} U_{pkq} \times a_{pq} \times b_k \times n_{kq}} \quad g^i_{pkq} = U_{pkq} \times a^i_{pq}, b^{i-1}_k, n^{i-1}_{kq},$$

$$T^i_k = \sum_{p\in P,\, q\in Q} g^i_{pkq} \quad T^i_{kq} = \sum_{p\in P} g^i_{pkq}.$$

Step 2. Stopping criteria

For every $k \in K$ and $q \in Q$, fix the error levels ε_k and ε_{kq}.

If $(|T^i_k - C_k| \le \varepsilon_k \wedge ||T^i_{kq} - F_{kq} \le \varepsilon_{kq}) \quad \vee \quad i = i_{\max}$,

then Stop.

Else, continue to Step 3.

Step 3. Updating

For every $p \in P$, $k \in K$and $q \in Q$, compute:

$$b^i_k = \min\left\{1, \frac{b^{i-1}_k \cdot C_k}{T^i_k}\right\} \quad n^i_{kq} = \min\left\{1, \frac{n^{i-1}_{kq} \cdot F_{kq}}{T^i_{kq}}\right\}.$$

Step 4. Continue

Set $i = i + 1$.

If $i + 1 \le i_{\max}$, repeat Steps 1 to 3.

Else, **Stop**.

The above algorithm is simple and can be encoded in any language. We used Visual Basic. To simplify the implementation of the computer code, we used an MS Excel file as program interface. In this way, the data could easily be input. The outputs were also reported in MS Excel. The computer hardware used was a PC with 2.33GHz Intel(R) Xeon(R) CPU and 3.25 GB of RAM.

4.3. Latent land values and shadow prices

There is a delicate interpretation of the dual variables β_k and θ_{kq} when considering the supply side. According to operational research terminology, β_k and θ_{kq} (beta and theta values) are the shadow prices associated with capacity and rationing constraints. With respect to the objective function, the shadow price β_k represents the value of one extra unit added to zone k. Consider two zones, k' and k'', with identical characteristics except for one: the units in zone k' have a lake view. Therefore, it is expected that $\beta_{k'} > \beta_{k''}$, as customers would compete to take the most attractive units. Similarly, it will be $\theta_{k'q} > \theta_{k''q}$. Thus, the shadow price is the price that the market is willing to pay for one additional unit satisfying certain characteristics.

Shadow prices can be treated as bidding prices since they are based on the marginal cost of adding one more unit to the supply side. It means that bidding transactions are the result of having more demand than supply and the occurrence of competition. Obviously, for low-demand units (excessive supply), no bidding transaction will take place, because there is no competition and hence possibly no transaction whatsoever. It is also likely that landlords will withdraw their units as they wait for the market to improve. In this case, the shadow price given by the model will be zero. Thus, a zero shadow price means there is not enough information to identify the price of respective properties.

Consequently, shadow prices are not real values, but rather relative variations of the values; they are usually called 'delta values'. Due to the complexity involved (Herath and Maier 2010; Rolon 2008), literature has lowered its expectation to digest relative real estate values (Rolon 2008).

Nonetheless, absolute values can be assumed to be values split between base value and delta value. That is, 'absolute value = base value + delta value'. Therefore, it can be stated that the land value is latent in the shadow price (delta value).

The primary objective of this study is to screen changes in land values (i.e. delta values) in response to changes concerning transportation infrastructures. Such an approach to the problem suits the need of identifying what is known as an 'affection plan' (that is, an official site plot plan issued by the government or authorized entities with survey coordinates delineating the boundary) with respect to drastic changes in transportation, such as new metro stations or new road corridors. Usually, zoning authorities are interested in seeing affection plans in terms of percentage changes with respect to certain prefixed percentage thresholds and regulations to enforce.

Our solution algorithm provides a simple way to identify the land values. The delta values are the outputs of this algorithm which show changes in property value due to externality factors like changes in transportation infrastructure. Base values can be regarded as the property values with no such externalities. There are two possible kind of predictions that can be made about property values: predictions due to changes in existing situations and predictions based on changes to happen in future situations.

For existing situations, the current value of a property can be assumed as the base value. Authorities usually monitor current transaction values, and ought to make such information publicly available. For future situations, the current value can simply be adjusted considering the annual inflation rates, which are also calculated and published by the authorities.

5. Numerical results

5.1. Preparing the case study

In this section, we report the analysis of a case study of the central business district (CBD) of the city of Winnipeg, Canada. The case study involved 100 customers, 10 location zones, and 100 land use types; thus:

$$|P| = 100, \quad |K| = 10, \quad |Q| = 100. \tag{29}$$

The location of the CBD and the zoning structure of the case study are shown in Figure 2. Below, we define and specify the case study so it can be used by other researchers as a benchmark.

As discussed above (see Figure 1), there are four sets of inputs:

$$\begin{aligned} &\{G_{pq}{:}p \in P,\ q \in Q\}, \quad \{U_{pkq}{:}p \in P,\ k \in K,\ q \in Q\}, \\ &\{C_k{:}\ k \in K\}, \quad \{F_{kq}{:}k \in K,\ q \in Q\}. \end{aligned} \tag{30}$$

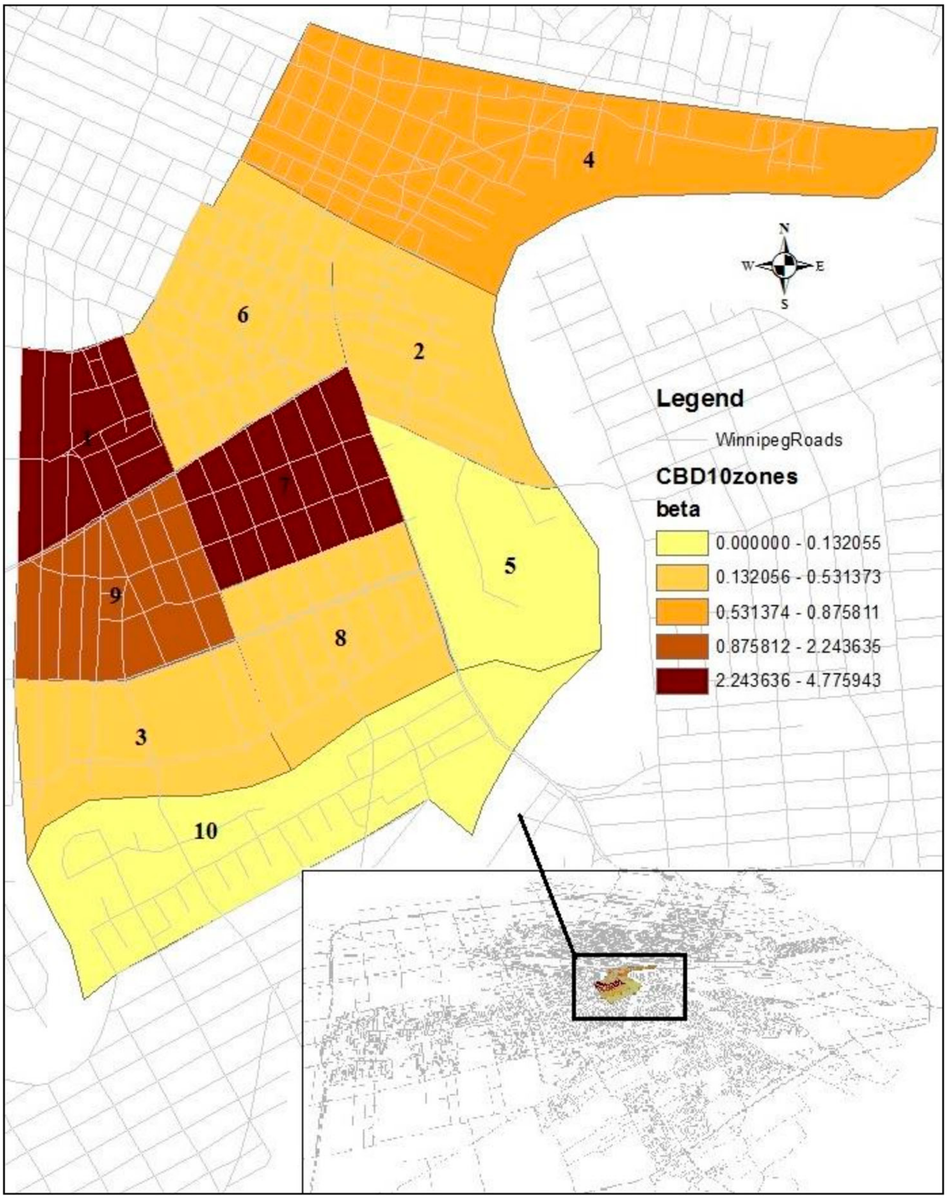


Figure 2. CBD and Winnipeg, Canada: Location and zoning structure.

These inputs are specified stochastically to avoid any bias to the results from applying the methodology to the case study. Tables 1 and 2 present the uniform random numbers used to compute the input data. With respect to the substitution made for Equations (23)–(25), the exponential format of the utility function U_{pkq} is defined as follows:

$$U_{pkq} = \exp\left(-R_{pU_{pkq}} \cdot R_{kU_{pkq}} \cdot R_{qU_{pkq}}\right). \tag{31}$$

That is, U_{pkq} depends both on the customer's preferences and the characteristics of the zone and land use type, represented by random numbers $R_{pU_{pkq}}$, $R_{kU_{pkq}}$, $R_{qU_{pkq}}$.

Table 1. Random numbers corresponding to customer and land use-type pairs (*p*,*q*).

p or *q*	R_{pGpq}	R_{qGpq}	R_{qFkq}	R_{pUpkq}	R_{qUpkq}	*p* or *q*	R_{pGpq}	R_{qGpq}	R_{qFkq}	R_{pUpkq}	R_{qUpkq}
1	0.856	0.312	0.412	0.759	0.76	51	0.872	0.813	0.23	0.911	0.991
2	0.744	0.794	0.129	0.776	0.481	52	0.402	0.473	0.596	0.981	0.835
3	0.859	0.177	0.862	0.974	0.784	53	0.502	0.287	0.349	0.465	0.784
4	0.519	0.173	0.104	0.943	0.778	54	0.222	0.702	0.563	0.77	0.784
5	0.641	0.546	0.992	0.33	0.667	55	0.158	0.374	0.834	0.758	0.979
6	0.928	0.749	0.146	0.973	0.973	56	0.94	0.829	0.488	0.991	0.825
7	0.06	0.487	0.437	0.964	0.939	57	0.153	0.574	0.559	0.648	0.079
8	0.06	0.316	0.77	0.982	0.498	58	0.488	0.862	0.694	0.534	0.762
9	0.454	0.5	0.262	0.666	0.978	59	0.403	0.477	0.883	0.368	0.549
10	0.492	0.495	0.935	0.412	0.376	60	0.861	0.417	0.134	0.399	0.575
11	0.024	0.62	0.803	0.665	0.996	61	0.914	0.869	0.063	0.647	0.794
12	0.136	0.58	0.809	0.92	0.961	62	0.642	0.37	0.014	0.211	0.46
13	0.035	0.243	0.529	0.61	0.756	63	0.087	0.913	0.538	0.573	0.593
14	0.539	0.385	0.258	0.778	0.72	64	0.989	0.638	0.846	0.791	0.963
15	0.764	0.468	0.951	0.51	0.739	65	0.032	0.74	0.436	0.95	0.893
16	0.886	0.436	0.7	0.145	0.31	66	0.937	0.938	0.875	0.981	0.979
17	0.209	0.37	0.714	0.181	0.465	67	0.002	0.118	0.2	0.974	0.52
18	0.616	0.94	0.408	0.744	0.84	68	0.543	0.347	0.31	0.73	0.782
19	0.624	0.206	0.929	0.937	0.878	69	0.988	0.147	0.386	0.299	0.608
20	0.531	0.067	0.117	0.602	0.163	70	0.51	0.236	0.299	0.297	0.714
21	0.766	0.456	0.619	0.712	0.968	71	0.743	0.025	0.881	0.998	0.991
22	0.196	0.129	0.486	0.618	0.935	72	0.153	0.125	0.055	0.953	0.891
23	0.608	0.081	0.909	0.925	0.984	73	0.608	0.639	0.552	0.341	0.299
24	0.264	0.015	0.621	0.997	0.998	74	0.079	0.232	0.12	0.723	0.812
25	0.982	0.49	0.184	0.644	0.96	75	0.146	0.744	0.725	0.745	0.98
26	0.962	0.464	0.567	0.561	0.909	76	0.775	0.874	0.317	0.846	0.972
27	0.967	0.872	0.313	0.973	0.365	77	0.715	0.869	0.855	0.95	0.689
28	0.914	0.08	0.002	0.673	0.985	78	0.075	0.755	0.128	0.999	0.996
29	0.947	0.852	0.364	0.921	0.911	79	0.64	0.758	0.767	0.958	0.96
30	0.76	0.402	0.381	0.962	0.967	80	0.407	0.663	0.382	0.574	0.842
31	0.396	0.783	0.541	0.463	0.986	81	0.152	0.132	0.272	0.891	0.937
32	0.595	0.717	0.482	0.998	0.897	82	0.808	0.616	0.931	0.815	0.304
33	0.892	0.851	0.844	0.9	0.395	83	0.621	0.446	0.049	0.939	0.627
34	0.319	0.214	0.387	0.888	0.346	84	0.587	0.887	0.233	0.949	0.355
35	0.651	0.916	0.729	0.976	0.96	85	0.112	0.571	0.949	0.395	0.978
36	0.214	0.268	0.154	0.805	0.937	86	0.861	0.602	0.267	0.619	0.517
37	0.297	0.432	0.569	0.229	0.817	87	0.53	0.803	0.361	0.571	0.651
38	0.579	0.85	0.452	0.893	0.662	88	0.65	0.981	0.781	0.724	0.804
39	0.481	0.854	0.34	0.451	0.785	89	0.125	0.473	0.019	0.415	0.817
40	0.657	0.323	0.325	0.632	0.88	90	0.994	0.122	0.344	0.823	0.561
41	0.72	0.943	0.196	0.981	0.967	91	0.038	0.538	0.488	0.387	0.86
42	0.405	0.026	0.24	0.706	0.894	92	0.709	0.989	0.606	0.584	0.885
43	0.8	0.468	0.169	0.84	0.537	93	0.275	0.344	0.339	0.677	0.975
44	0.387	0.862	0.99	0.742	0.791	94	0.217	0.289	0.277	0.332	0.409
45	0.079	0.41	0.331	0.865	0.88	95	0.139	0.324	0.898	0.991	0.923
46	0.856	0.451	0.183	0.948	0.987	96	0.322	0.985	0.352	0.935	0.879
47	0.547	0.608	0.851	0.883	0.646	97	0.086	0.966	0.543	0.926	0.961
48	0.475	0.636	0.316	0.882	0.987	98	0.803	0.474	0.246	0.943	0.937
49	0.901	0.932	0.925	0.573	0.403	99	0.397	0.609	0.394	0.389	0.92
50	0.532	0.121	0.793	0.634	0.715	100	0.137	0.962	0.883	0.931	0.978

Characteristics impacting the customer preferences, such as socio-economics, job, gender, and other demographic factors, are represented by $R_{pU_{pkq}}$. Also, $R_{kU_{pkq}}$ and $R_{qU_{pkq}}$ include features of zones and land use types such as zoning regulations, sales/rent prices, accessibility, security, and neighbouring amenities.

Note that we could have specified utilities based on a single random term like $U_{pkq} = \exp(-R_{pkqU_{pkq}})$ which must return 10,000 records of $R_{pkqU_{pkq}}$ to populate the case study. Instead, to establish the case study in a concise format and imitate a real

Table 2. Random numbers and inputs corresponding to the zones (k).

	Inputs					Outputs of the last iteration $i_{max} = 100$		
k	R_{kC_k}	$R_{kF_{kq}}$	$R_{kU_{pkq}}$	C_k, capacity of zone k	Sum of all F_{kq} on q	Shadow price of C_k $b_k = e^{-\beta_k}$	Error; $\lvert C_k - T_k \rvert$	T_k, transactions in zone k
1	0.811	0.182	0.888	2883.2	24,260.5	0.0084	34.11	2917.35
2	0.269	0.86	0.047	18,092.8	18,092.9	0.6257	3.15	18,095.94
3	0.315	0.728	0.561	15,889.2	15,889.3	0.6429	6.68	15,895.90
4	0.074	0.032	0.64	35,943.0	41,578.1	0.4165	163.91	36,106.87
5	0.135	0.736	0.577	27,585.2	27,585.4	1.0000	365.23	27,219.99
6	0.208	0.569	0.403	21,621.3	21,621.4	0.6762	6.90	21,628.15
7	0.568	0.037	0.126	7779.8	40,030.1	0.0146	92.06	7871.87
8	0.367	0.786	0.541	13,812.2	13,812.3	0.5878	4.36	13,816.60
9	0.53	0.56	0.539	8747.0	13,147.8	0.1061	40.36	8787.37
10	0.088	0.508	0.207	33,436.8	33,437.0	0.8763	13.70	33,450.52
Total				185,790.56[a]	249,454.7		730.44	185,790.55

[a]Total demand (sum of all G_{pq} on p and q) is 185,790.55.

situation, we have used the product of three random terms reported in Tables 1 and 2 (2×100 of these random terms are in Table 1; 10 of them are in Table 2). The same rationale is applied when defining the other inputs.

To simulate a real situation on the demand (G_{pq}) and the supply sides (C_k, F_{kq}), we employed random numbers in the exponential form, which yield a highly unpredictable dataset (Bagloee Asadi and Reddick 2011; Leemis and Park 2006). An exponential random number x with an expected value of λ can be simply computed as follows:

$$x = -\lambda \ln (R), \tag{32}$$

where $0 < R \leq 1$ is a uniform random number. Thus, the demand and supply inputs can be assumed to be as follows:

$$G_{pq} = 10 \ln (R_{pG_{pq}} \times R_{qG_{pq}}), \tag{33}$$

$$C_k = -10,000 \ln (R_{kC_k}), \tag{34}$$

$$F_{kq} = -100 \ln (R_{kF_{kq}} \times R_{qF_{kq}}). \tag{35}$$

In Equation (33), we assumed 10 transactions as the expected demand associated with the pair (p, q), but this number will vary depending on two random numbers $R_{pG_{pq}}$, $R_{qG_{pq}}$ representing the limit values for the demand. The total demand becomes $\sum_{pq} G_{pq} = 185,790$ transactions. In Equation (34), given the size of the demand matrix (100×100) and assuming 10 as the expected value of an entry and 10 units as the expected capacity (number of units per zone) at all zones, in order to meet the demand we must have $E(C_k) = 10,000 = 100 \times 100 \times 10/10$). Also, the capacities vary across the zones due to the random number R_{kC_k}. In Equation (35), assuming 10 zones each of them with an expected capacity of 10,000 units and 100 land use types, the expected rate for land rationing to process the demand must be $E(F_{kq}) = 100 = 10,000 \times 10/(10 \times 100)$. Again, the land rationing varies due to the two random numbers representing the corresponding limit values.

To ensure the feasibility of the solution, the set of constraints consisting of Equations (9), (14), and (15) must be satisfied. Therefore, the following simple adjustment was made on the computed input rates in Equations (33)–(35) (with the symbol := meaning 'set to'):

$$C_k := C_k \frac{\sum_{p\in P,q\in Q} G_{pq}}{\sum_{k\in K} C_k}, \tag{36}$$

$$F_{kq} := F_{kq} \frac{\sum_{p\in P} G_{pq}}{\sum_{k\in K} F_{kq}}, \tag{37}$$

$$F_{kq} := F_{kq} \frac{C_k}{\sum_{q\in Q} F_{kq}}, \quad \text{if } \frac{C_k}{\sum_{q\in Q} F_{kq}} > 1. \tag{38}$$

In Equations (36) and (37), the sum of the capacities of all the zones $\left(\sum_k C_k\right)$and the sum of the rates of land rationings $\left(\sum_k F_{kq}\right)$ are balanced to total demand $\left(\sum_{pq} G_{pq}\right)$. In Equation (38), the rationing volumes are adjusted up to the capacity of the corresponding zone, to eliminate the chance of bottlenecks in the outbound flow. Table 2 indicates the final values of all capacities of all zones and the aggregate land rationing rate.

5.2. Executing the algorithm

The algorithm described in Section 4 has been applied to the data developed for the case study above with only one stopping criterion, namely, the one given by the maximum number of iterations $i_{\max} = 100$. As shown in Figure 1, given the four sets of input variables in Equation (30), the algorithm yields three sets of output variables which are:

(i) g_{pkq}, transaction made by customer p interested in land use type q in zone k;
(ii) $b_k = \mathrm{e}^{-\beta_k}$, shadow price of the capacity provided for zone k; and
(iii) $n_{kq} = \mathrm{e}^{-\theta_{kq}}$, shadow price of the land rationing rate in zone k for land use type q.

Given the transaction volume g_{pkq}, it is easy to calculate the total number of transactions T_k made in zone k and the total number of units T_{kq} of land use type q that were transacted in zone k. In other words, we have: $T_k = \sum_{p\in P,q\in Q} g_{pkq}$ and $T_{kq} = \sum_{p\in P} g_{pkq}$. (See the Solution Algorithm Outline in Section 4.2.)

Table 2 shows the transaction volumes T_k for each zone k computed for the last iteration and the corresponding shadow prices. Moreover, the variation of beta for the CBD is also graphically shown in Figure 2.

Table 3 presents the aggregate gaps at each iteration i between transaction volumes and capacity (i.e. $\sum_{k\in K} |C_k - T_k^i|$), as well as those between transaction volumes and land use rationing rates (i.e. $\sum_{k\in K,q\in Q} |F_{kq} - T_{kq}^i|$) for 100 successive iterations. The target values obtained at the last iteration ($i_{\max} = 100$) are indicated by **.

Table 3 shows that the values of the gaps decrease from one iteration to the next one. This witnesses the algorithm convergence behaviour. Furthermore, an error index based on the gap between zone capacity and transaction is defined in Table 3 for each iteration i as $\%\mathrm{Err}(k) = \sum_{k\in K} |C_k - T_k^i| / \sum_{k\in K} C_k$. Figure 3 graphically illustrates the rapid decrease of the error index over 100 successive iterations.

As shown in Table 3 and Figure 3, after only four iterations and 1.63 minutes, the percentage error falls below 5%. While the running time for the entire 100 iterations is 36.15

Table 3. Algorithm convergence results for 100 successive iterations.

i = Itr	$\sum_{k\in K}\lvert C_k - T_k^i\rvert$	$\sum_{k\in K,q\in Q}\lvert F_{kq} - T_{kq}^i\rvert$	%Err_k	Time (min)	i = Itr	$\sum_{k\in K}\lvert C_k - T_k^i\rvert$	$\sum_{k\in K,q\in Q}\lvert F_{kq} - T_{kq}^i\rvert$	%Err_k	Time (min)
1	89,840.1	98,914.2	48.36%	0.35	51	1364.5	67,151.8	0.73%	16.23
2	33,680.5	98,822.6	18.13%	0.68	52	1337.2	67,105.8	0.72%	16.55
3	20,022.0	86,017.7	10.78%	1.00	53	1323.4	67,058.6	0.71%	16.87
4	11,357.4	82,032.7	6.11%	1.30	54	1296.2	67,014.0	0.70%	17.17
5	8261.8	77,170.0	4.45%	1.63	55	1283.8	66,968.2	0.69%	17.48
6	7877.1	76,534.6	4.24%	1.97	56	1258.8	66,925.1	0.68%	17.80
7	7027.6	75,475.1	3.78%	2.27	57	1245.7	66,882.5	0.67%	18.10
8	6455.0	74,640.9	3.47%	2.60	58	1222.8	66,844.1	0.66%	18.45
9	5896.8	74,155.3	3.17%	2.90	59	1210.3	66,805.2	0.65%	18.75
10	5397.9	73,463.9	2.91%	3.22	60	1190.5	66,768.4	0.64%	19.07
11	5029.7	73,073.7	2.71%	3.55	61	1176.7	66,731.1	0.63%	19.40
12	4604.6	72,524.4	2.48%	3.87	62	1158.7	66,695.5	0.62%	19.80
13	4391.9	72,198.4	2.36%	4.17	63	1144.0	66,660.0	0.62%	20.23
14	4016.2	71,753.5	2.16%	4.50	64	1127.4	66,627.3	0.61%	20.60
15	3891.1	71,484.1	2.09%	4.82	65	1111.5	66,593.4	0.60%	21.05
16	3560.8	71,115.7	1.92%	5.12	66	1096.0	66,560.8	0.59%	21.50
17	3491.7	70,915.1	1.88%	5.45	67	1080.4	66,527.6	0.58%	21.93
18	3197.4	70,610.5	1.72%	5.75	68	1065.3	66,495.9	0.57%	22.37
19	3153.5	70,422.0	1.70%	6.07	69	1050.2	66,465.8	0.57%	22.80
20	2901.4	70,158.3	1.56%	6.38	70	1037.1	66,437.3	0.56%	23.23
21	2870.6	69,999.2	1.55%	6.70	71	1024.3	66,408.0	0.55%	23.67
22	2682.0	69,777.4	1.44%	7.00	72	1011.8	66,379.9	0.54%	24.10
23	2641.8	69,636.3	1.42%	7.32	73	999.3	66,351.7	0.54%	24.53
24	2500.8	69,445.6	1.35%	7.67	74	987.2	66,325.2	0.53%	24.97
25	2453.9	69,324.1	1.32%	7.97	75	974.9	66,299.2	0.52%	25.38
26	2346.6	69,146.4	1.26%	8.28	76	962.7	66,274.5	0.52%	25.82
27	2290.3	69,034.3	1.23%	8.63	77	951.0	66,250.5	0.51%	26.27
28	2211.5	68,889.0	1.19%	8.93	78	939.9	66,227.2	0.51%	26.70
29	2152.9	68,798.3	1.16%	9.25	79	928.7	66,204.9	0.50%	27.13
30	2097.5	68,667.9	1.13%	9.58	80	917.5	66,182.4	0.49%	27.57
31	2034.9	68,583.5	1.10%	9.88	81	906.2	66,160.0	0.49%	28.00
32	2001.4	68,466.1	1.08%	10.20	82	894.9	66,137.6	0.48%	28.43
33	1935.2	68,382.5	1.04%	10.53	83	883.8	66,115.7	0.48%	28.87
34	1913.9	68,278.0	1.03%	10.85	84	872.7	66,093.8	0.47%	29.30
35	1854.2	68,197.6	1.00%	11.15	85	861.8	66,072.4	0.46%	29.75
36	1830.5	68,101.6	0.99%	11.50	86	851.0	66,051.0	0.46%	30.18
37	1777.3	68,024.0	0.96%	11.80	87	840.3	66,030.1	0.45%	30.62
38	1750.8	67,935.6	0.94%	12.12	88	829.9	66,010.2	0.45%	31.05

39	1703.2	67,862.8	0.92%	12.45	89	820.2	65,991.2	0.44%	31.48
40	1674.6	67,782.4	0.90%	12.75	90	811.1	65,972.3	0.44%	31.93
41	1631.6	67,714.8	0.88%	13.05	91	802.1	65,953.8	0.43%	32.37
42	1603.3	67,645.8	0.86%	13.38	92	793.3	65,935.0	0.43%	32.78
43	1568.2	67,586.0	0.84%	13.68	93	784.5	65,917.1	0.42%	33.22
44	1542.1	67,523.9	0.83%	14.00	94	775.9	65,899.0	0.42%	33.58
45	1512.3	67,465.3	0.81%	14.32	95	767.3	65,881.9	0.41%	34.03
46	1485.3	67,407.9	0.80%	14.65	96	759.1	65,865.5	0.41%	34.48
47	1459.4	67,351.6	0.79%	14.97	97	751.6	65,850.8	0.40%	34.90
48	1430.6	67,297.6	0.77%	15.28	98	744.6	65,836.1	0.40%	35.32
49	1408.1	67,247.3	0.76%	15.60	99	737.5	65,821.6	0.40%	35.73
50	1380.6	67,200.2	0.74%	15.92	100	730.4**	65,806.7**	0.39%	36.15

Note: Itr = Iteration number. C_k and T_k are the available units in the market and the transaction volume in zone k. F_{kq} and T_{kq} are the rationing rate and corresponding transactions of land use type q in zone k; $\%Err\_k = \sum_k |C_k - T_k| / \sum_k C_k$.

**Target values for $\sum_k |C_k - T_k|$ and $\sum_{k,q} |F_{kq} - T_{kq}|$ are 0.01 and 63,664.13, respectively.

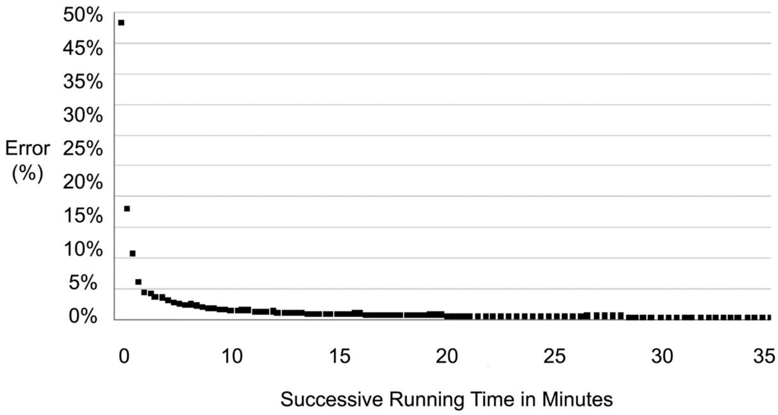


Figure 3. Algorithm convergence over 100 successive iterations.

minutes, the process can be terminated earlier when a satisfactory solution is obtained. For instance, at iteration 35 after only 11.15 minutes, the error index falls below 1%, which is, empirically, an acceptable accuracy in transportation practice (INRO 2009).

It is worth noting that the way the case study was populated results in more computation time than would likely be required in an actual application. In reality, not all zones would be considered by customers. The available zones can be filtered and so considerably reduce both the case size and computation time.

5.3. Interpretation of the algorithm's outputs

There are two sets of shadow prices related to the supply side:

$$\{b_k{:}k \in K\} \quad and \quad \{n_{kq}{:}k \in K, q \in Q\}.$$

Table 2 presents shadow prices pertaining to zone capacities in exponential form, that is, $b_k = e^{-\beta_k}$. It is clear that after 100 iterations, despite having the total of available units in the market (capacity of all the zones) equal to the demand, not all units in Zone 5 have been transacted. Zone 5 shows a shadow price of 0 (in exponential form, $b_5 = 1$), a capacity of 27,585 units, and transacted volumes of only 27,220 units. For the remaining zones, we have $0 < b_k < 1$ or $0 < \beta_k < \infty$. A smaller value for b_k indicates a higher shadow price β_k, which implies that the corresponding zone is more attractive to customers and, consequently, the competition to possess the units within it is stronger than in other zones. This interpretation of the beta values provides some key insights on a variety of questions, such as:

- What zones present a relevant shortage of units? These zones can be identified as those areas for which the beta value is high.
- How many additional units should be supplied per zone? Extra units can be added to a zone whose beta value is greater than zero until the beta reaches zero.
- How can location attractiveness be adjusted and increased through changing zone attractiveness characteristics? This issue is of great interest to zoning authorities:

housing or municipality fees can be managed to balance the demand among the zones. Note, in particular, that increasing accessibility may compensate for other disadvantages and increase the attractiveness of a zone.

- What is the true (not bubble) maximum value of a unit? The asking price fed into the Logit land use choice model can be adjusted until the beta values for the zones become equal. As a consequence, all zones can be valued on the basis of their relative values to customers. This also implies that the maximum value of a unit cannot be higher than the one determined for the zone where it belongs. Assigning a value to units higher than this maximum is clearly unrealistic and creates bubble prices.
- Is the city zoning system operating efficiently? If the beta values across all the zones are equal, then the system is operating efficiently. The degree to which the beta values are unequal indicates the extent of the system's inefficiency. A beta value greater than zero indicates that there is a unit shortage. A beta value equal to zero means there is a unit surplus.

Similarly, with respect to the shadow price of land use rationing θ_{kq} (theta value), the following issues arise:

- Is the current rationing system – that is, designating units for specific land uses – efficient? The degree to which the rationing is efficient can be evaluated by measuring how close to be equal the theta values are.
- How many units can be reserved for specific land use types? The number of designated units can be adjusted as long as the theta values across different land uses remain equal.
- Given an underutilized land use type within a zone, how many units can be reclaimed for other land uses? The answer is that one can reclaim units destined to a particular land use type until the theta value becomes non-zero.
- What is a fair property tax for different land use types in a zone? Property taxes can be adjusted so that the theta values across different land use types become equal.

5.4. Transportation infrastructures

Transportation infrastructures may have a profound impact on land use choices from many different viewpoints involving both positive and negative aspects (Gim 2013), all of which can be considered in the utility function associated with the possible choices. Changes in the choices would reflect on the property value which is in turn captured by shadow price changes.

The primary aim of transportation is to provide accessibility, and accessibility is the most significant positive factor to evaluate a given piece of land. The most direct proxy for accessibility is travel time, which can easily be specified in transportation modelling/planning. Therefore, travel time to major employment or residential centres can be considered as one explanatory variable in the utility functions. Any change in the transportation infrastructure, such as constructing a new metro lines, working to improve roads or highway corridors, and even blocking a road for construction would affect the travel time and, hence, the utility from choosing a certain land use and the corresponding property value.

Our methodology shows the necessity for the authorities in charge of transportation planning, funding, and zoning to make more informed decisions. In order to make such decisions, authority committees need to know where and to what extent the property value will increase due to a given investment in the transportation infrastructure. A portion of such windfall value can be then returned to the public via a property tax – VC – for further investment.

Also, our methodology can be utilized in TOD, where it is necessary to know where and to what extent accessibility would increase. Hence, building highly demanded developments such as commercial centres within prime areas would add value to the investment due to the premium accessibility. One of the key points in TOD is to identify the type and quantity of the land uses surrounding a transit centre such that the value of the land uses in the market makes the investment viable.

Our methodology can be combined with a transportation planning model to obtain travel time estimations, as well as some adverse factors evaluations, such as pollutant and noise levels.

6. Summary and conclusions

We have developed a model for land use choices where the factors contributing to customers' decisions are formalized using the logit model. Customers make their choices in competition with each other, which is coded in a mathematical programming framework. Our model explicitly considered the available number of units in the market, as well as the zoning restrictions on rationing the land to specific land uses.

We considered both the sizes of the locations/zones (known as capacities) and the land use rationing (or zoning) constraints explicitly in the mathematical programming problem. The introduction of LRCs is required to cover real life situations where zoning authorities enforce regulations or restrictions for land use planning. Despite the fact that these constraints clearly make the model more realistic, they have been ignored in the literature due to computational complexities associated with them. Therefore, integrating LRCs actually constitutes the unique aspect of the proposed study.

A solution algorithm using the SCD was developed for the logit-based mathematical programming. The algorithm was tested on an artificial and challenging benchmark. The results show the algorithm convergence.

Transportation-related factors significantly shape land use choices and the real estate market. Changes in such choices are reflected in the property value. The primary objective of this study was to screen changes in the land value in response to changes in the transportation infrastructure. The model we developed has produced some by-products that can suit such a purpose. The mathematical programming model yields shadow prices on the supply side, which can be interpreted as the unit values in the market. Therefore, changes in land values can be described in terms of changes in the shadow prices.

Changes to transportation infrastructure may have positive or negative impacts on property values, which can be accounted for in the utility function associated with the available choices. The primary aim of transportation is to provide accessibility that can be introduced and measured as travel time. The methodology developed in this study is theoretically able to capture such cascading interactions existing between transportation and land use value. The proposed methodology can be used by the authorities in charge of transportation planning, funding, and zoning as a means to make more informed decisions, in particular, those concerning the application of VC taxes and TOD plans.

Given the interaction between land use, land value, and accessibility, there are three main lines for research that can be envisaged for further investigations:

(i) In network design problems where investments in enhancing accessibility are studied (Bagloee Asadi, Tavana, et al. 2013), an added-value term for the affected properties can be included into the objective function. In other words, investments can be further boosted by the concept of VC.
(ii) The Braess paradox (Braess 1968; Braess, Nagurney, and Wakolbinger 2005) provides an interesting angle on the connection between land use and transportation. It states that there might exist 'infected' roads, that is, some roads that, if closed, would increase the accessibility and mobility of the entire network. According to Braess, such infected roads should be turned to different land uses (Bagloee Asadi, Ceder, et al. 2013). In the same spirit, Bagloee Asadi and Ceder (2011) state that such dead roads could be dedicated to transit centres so as to also enhance accessibility. Consequently, the notion of TOD can be further considered as an option.
(iii) In large scale development projects at both sides of the equation (land use and transportation), projects prioritizations targeting at achieving maximum level of accessibility or highest amount of property price is of paramount importance (Bagloee Asadi and Asadi 2015; Bagloee Asadi and Tavana 2012). As such, a property developer in charge of a massive and (perhaps) phased construction has to align construction to future changes in the road network. On the other hand, road authorities might need to prioritize road improvement projects in order to cater for some upcoming monumental developments.

Disclosure statement

No potential conflict of interest was reported by the authors.

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