



An investigation of the robustness in the Travelling Salesman problem routes using special structured matrices

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ABSTRACT

In this study, the robustness of the Travelling Salesman Problem (TSP) routes is investigated by recognising the special combinatorial structures of Kalmanson matrices. A recognition algorithm encompassing three procedures based on combinatorial and linear programming (LP) is developed and executed on several randomly generated instances. These procedures produce three lower bounds which provide guarantees on the optimality of the solutions. Computational experiments show that the proposed LP-based procedure performs efficiently well across all problem dimensions and provides the best lower bounds to the TSP. This is supported by an average deviation of less than 7% between the TSP tour lengths and the lower bounds of the Kalmanson matrices.

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1. Introduction

The Travelling Salesman Problem (TSP) is a famous classical NP-Hard problem which has been extensively studied for over half a century since it was first formulated mathematically in the 1930s (Hahn, 1932). Given a number of cities and the cost of travel between them such as distance or time, the TSP is defined as the problem of designing the shortest vehicle route which starts from a city, passes through all other cities and in the end, returns to the home city. Each city must be visited exactly once by a single unbounded capacity vehicle (Gilmore, Lawler, Shmoys, & Lawler, 1985; Gutin & Punnen, 2006). More formally, the TSP is concerned with determining a Hamiltonian tour for an undirected graph such that the total cost is minimum.

Nonetheless, the tours of routing problems are known to be exposed to uncertain and unpredictable environments and thus vulnerable to changes. These uncertainties include traffic jams, accidents, weather condition and customer availability. In a real-life delivery problem, for instance, situations such as only a subset of n clients needs to be considered on a particular day or the unavailability of prior information on the number of customers to be served, are common. Apparently, this may cause the current routing plan to be no longer optimal or lose its

robustness. Due to this reason, achieving a robust routing plan that can cope with such unpredictable situations is a significant challenge to maintain the optimality of the tour.

A typical approach for dealing with this situation is to re-optimize the routing schedule for the subset. Nevertheless, this procedure may be expensive, challenging and time-consuming if it is performed on a frequent basis or if the number of clients in the subset is large, in which only slight manual adjustment would be necessary. The re-optimization exercise also is almost impossible if there is a lack of information on the clients' subset in advance. Alternatively, the problem can be solved by constructing an advance tour for n clients, and then the subset of clients for the day is served according to the same ordering as they appear in the advance tour. In other words, the sub-tour for the subset of clients is induced from the advance tour by skipping the missing clients (Schalekamp & Shmoys, 2008).

The problem can be considered as an example of the universal and a priori TSP. In both cases, the problem is generally concerned with finding an advance tour which could perform well on any subsets of clients (Gorodezky, Kleinberg, Shmoys, & Spencer, 2010) in the sense that the sub-tour is close to the optimal tour (Hajiaghayi,

Kleinberg, & Leighton, 2006). The *universal TSP* was first studied by Bartholdi and Platzman (1982) in which the space-filling curve heuristic with a competitive ratio of $O(N \log N)$ has been proposed for solving the planar TSP problem. In this problem, the ratio of the 'length of the induced sub-tour to the length of the optimal tour' for all potential clients' subsets is computed (Schalekamp, 2007). The maximum ratio over all subsets, known also as a competitive ratio ρ for the advance tour guarantees that the length of the induced sub-tour is at most ρ times the length of the optimal tour (Gorodezky et al., 2010). The subset of this ratio is selected adversarially and the problem aims to find a universal TSP tour such that the competitive ratio over all possible advance tours is minimised. On the other hand, the study on an a priori TSP introduced by Jaillet (1985) deals with a probabilistic model. In this case, the clients' subsets are assumed to have a known probability distribution. The problem of constructing an a priori TSP tour through all clients with respect to minimising the 'expected length of the induced sub-tour' (Schalekamp & Shmoys, 2008) is defined as a Probabilistic TSP (PTSP) (Jaillet, 1985).

In early studies, exact classical approaches such as the branch and cut algorithm by Karaođlan and Kesen (2017) and Laporte, Louveaux, and Mercure (1994) and the branch and bound algorithm by Amar, Khaznaji, and Bellalouna (2017), Mahfoudh, Khaznaji, and Bellalouna (2015), and Berman and Simchi-Levi (1988) which could guarantee the optimality of the solutions, were adopted for solving the PTSP. One should note that many computational analyses based on these approaches are only capable of solving small problem instances. Consequently, in order to handle larger problem sizes, efficient heuristic algorithms, as well as tour construction and local search procedures, are presented in Rossi and Gavioli (1987), Bertsimas and Howell (1993), Bianchi, Knowles, and Bowler (2005), Birattari, Balaprakash, Stützle, and Dorigo (2008), and Weyland, Montemanni, and Gambardella (2013).

Recently, the metaheuristic algorithms such as the Genetic Algorithm (Ding, Cheng, & He, 2007; Hui, 2012; Liu, 2008; Liu, 2010; Liu & Zeng, 2009; Maity, Roy, & Maiti, 2017), Simulated Annealing (Balaprakash, Birattari, Stützle, & Dorigo, 2007; Bowler, Fink, & Ball, 2003; Ezugwu, Adewumi, & Frincu, 2017; Lin, Bian, & Liu, 2016; Meer, 2007) and Ant Colony Optimization (Balaprakash, Birattari, Stützle, Yuan, & Dorigo, 2009; Bianchi & Dorigo, 2006; Bianchi & Gambardella, 2007; Bianchi, Gambardella, & Dorigo, 2002; Birattari, Balaprakash, & Dorigo, 2005; Branke & Guntsch, 2004; Eldem & Ülker, 2017; Hui, 2012; Zhou, He, Hou, & Qiu, 2018) in finding the suboptimal solutions for the PTSP

have emerged. A recent study by Smith and Chen (2016) presented a new meta-heuristic approach called Genetic Minimum Matrix Search (GMMS) for solving the PTSP. This evolutionary algorithm was tested on benchmark problems from TSPLIB, based on which the comparison results obtained by employing other meta-heuristic approaches such as Tabu Search and Ant Colony Optimization showed that 9 new best solutions were found out of 20 cases. In addition, some other algorithms as well as the memetic algorithm (Balaprakash et al., 2007; Castro, Sörensen, Vansteenwegen, & Goos, 2013; Kóczy, Földesi, & Tüü-Szabó, 2018; Lu, Benlic, & Wu, 2018; Wang, Chen, & Lin, 2017) and scatter search (Liu, 2007) have been presented in previous studies.

Over the last two decades, a number of special cases of the TSP that can be solved easily and efficiently using polynomial-time algorithms have been recognised (Burkard, Deineko, van Dal, van der Veen, & Woeginger, 1998; Gilmore et al., 1985). Using these special cases, one can approximate a good solution to the TSP instances (Baki & Kabadi, 1998). The special cases, known as polynomially testable cases of TSP, contain special combinatorial structures which can be identified by examining the underlying distance matrix of the TSP (Burkard et al., 1998; Gilmore et al., 1985).

This paper aims to investigate the robustness of the routing tours using the recognition of special structured matrices. In particular, the robustness of TSP routes is analysed empirically by applying several theoretical concepts of TSP in real-life instances, based on which their applicability is investigated. The motivation of this study stems from the aspiration of decision makers at Coventry City Council to obtain better routing and scheduling plans for their services, especially the meals delivery and incontinence laundry services which are initially scheduled manually (Aziz, Quah, & Mohammad, 2016). In this study, a robust routing plan is defined as a routing strategy that can cope with reductions in the number of customers, i.e. the tour is still optimal if one simply skips the customers that are removed from the service. In addition, the findings of this study should be able to identify which procedure provides better lower bounds to the TSP solutions, estimate how far the TSP tour lengths are from the lower bounds and examine the amount of reduction needed for a random TSP matrix to sufficiently satisfy the conditions of the special structured matrices.

The rest of the article is organised as follows. In the next section, we first define relevant theoretical concepts. Section 3 presents the proposed methodology followed by the computational results in Section 4. Finally, we offer some concluding remarks in Section 5.

2. Theoretical concepts

In this study, the robustness of TSP routes is analysed using the implementation of three procedures in which the definitions and theorems proposed by (Deineko, Rudolf, & Woeginger, 1995) are applied. The fundamental concept underlying these procedures known as the *master tour* was first formulated by Papadimitriou (2003).

2.1. The master tour

A master tour can be defined as follows: In an optimal tour, π of a problem, X is called a master tour if after removing any subset of points in X , π remains optimal. To illustrate, consider the example shown in Figure 1.

The illustration above shows a set of customers receiving a particular service scattered over seven locations. The optimal TSP tour for Figure 1(a) is $\pi' = 1, 2, 3, 4, 5, 6, 7, 1$ and π is also a master tour. Suppose that after few years, Customers 3 and 6 leave the service. Hence, the new optimal tour for Figure 1(b) can be obtained by simply removing these two customers from π , that is $\pi' = 1, 2, 4, 5, 7, 1$.

The notion of the master tour was first coined by Papadimitriou (2003). Following this definition, it results that the master tour is closely related to the universal TSP tour in the sense that all possible sub-tours for the clients' subsets induced in the universal TSP tour are indeed optimal. Hence, a master tour is, in fact, a universal TSP tour with a ratio of 1 (Schalekamp & Shmoys, 2008). Several studies in the literature have discovered that the master tour property has a close relationship with special cases of the TSP, specifically the Kalmanson matrix (Christopher, Farach, & Trick, 1996; Deineko et al., 1995).

As such, recognising a master tour for a particular TSP instance could guarantee the robustness and optimality of a tour regardless of any changes, hence leading to costs, distance and time savings. This concept may be useful in practice especially for services with daily operations where the routing and scheduling design do not need to be performed every day or frequently.

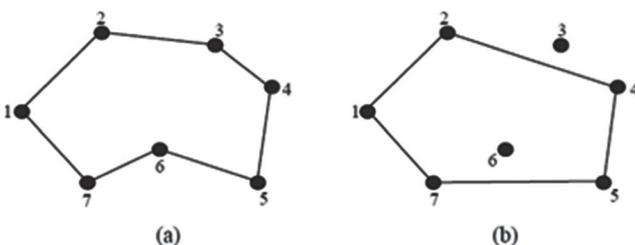


Figure 1. An example to illustrate the concept of the master tour.

2.2. Kalmanson matrices

In 1956, Flood (1956) showed that the minimal tour does not intersect itself. This means that for any convex quadrangle, the total length of two intersecting edges is always at least the same as the total length of two opposite sides (Woeginger, 2002). Hence, the shortest tour of the TSP can easily be determined by calculating the total length of the boundary of the convex polygon. However, in 1975, Kalmanson (1975) revealed that this observation was more related to the structures of the distance matrix instead of the convexity of the TSP. As such, the following definition was introduced.

Definition: A symmetric $n \times n$ matrix C is a *Kalmanson matrix* (or simply *Kalmanson*) if it fulfils the following conditions:

$$c_{i,l} + c_{j,k} \leq c_{i,k} + c_{j,l} \quad \text{for all } 1 \leq i < j < k < l \leq n \quad (1)$$

$$c_{i,j} + c_{k,l} \leq c_{i,k} + c_{j,l} \quad \text{for all } 1 \leq i < j < k < l \leq n \quad (2)$$

Deineko et al. (1995) then reduced the complexity of these conditions to $O(n^2)$, referred to as the *Kalmanson conditions* hereafter, as follows:

$$c_{i,j+1} + c_{i+1,j} \leq c_{i,j} + c_{i+1,j+1} \quad \text{for all } 1 \leq i \leq n-3, i+2 \leq j \leq n-1 \quad (3)$$

$$c_{i,1} + c_{i+1,n} \leq c_{i,n} + c_{i+1,1} \quad \text{for all } 2 \leq i \leq n-2 \quad (4)$$

The Kalmanson matrix is a special case of the TSP and the identity permutation of the Kalmanson matrix $1, 2, \dots, n-1, n$ always solves the TSP to optimality (Kalmanson, 1975). This class of matrix plays a crucial role in combinatorial optimisation problems as the solution of the TSP can be determined efficiently by solving the problem in polynomial time if the underlying distance matrix is a Kalmanson matrix. The special combinatorial structures of a Kalmanson matrix can be identified by examining the distance matrix of the TSP. While this completely depends on how the cities are numbered, specifying an appropriate numbering approach is crucial to transforming the TSP distance matrix into a Kalmanson matrix. Indeed, any cyclic ordering of this tour that is by rotation or reversion will yield a Kalmanson matrix (Baki & Kabadi, 1998; Deineko et al., 1995). Deineko et al. (1995) presented the relationship between the Kalmanson matrix and master tour, using the following theorems:

Theorem 1: A symmetric $n \times n$ matrix C possesses the *master tour* if and only if C is a *Kalmanson matrix*.

Theorem 2: For a symmetric $n \times n$ matrix C , it can be decided in $O(n^2 \log n)$ time whether C is a permuted Kalmanson matrix.

Theorem 3: For a symmetric $n \times n$ matrix C , it can be decided in $O(n^2 \log n)$ time whether C possesses a master tour.

Then, the $O(n^2 \log n)$ time complexity has been improved by Christopher et al. (1996) to $O(n^2)$ time.

The theorems represented in this section clearly demonstrate that the concept of a master tour is closely related to the Kalmanson matrices. To be clarified, recognising these special structures in a TSP distance matrix will facilitate finding a master tour which can guarantee the robustness and optimality of the TSP solution.

3. Methodology

In this study, we utilise a pre-generated distance matrix formed from 130 postcodes to perform the robustness testing procedures. The postcodes were randomly generated using Microsoft's Visual Basic for Applications (VBA) in Microsoft® Excel 2007 (or simply Excel) in conjunction with Microsoft® MapPoint 2009 (MapPoint). The distance between each pair of postcodes in this matrix is estimated using the MapPoint software. Here, it is tried that the generated postcodes represent real and valid points in the regions of Coventry, a city and local government district in the metropolitan county of West Midlands in England. As mentioned earlier, this study aims to empirically examine the robustness of the TSP routes on real-life instances, which are based on the services provided by the Coventry City Council, the biggest organisation which provides local government services in the city of Coventry.

From these 130 postcodes, we randomly generate various sets of test problems of different dimension (problem size) ranging from five to fifty postcodes using VBA. For each dimension, 30 instances with different lists of postcodes are randomly generated to observe the trends. Thus, a total of 1380 random instances across 46 dimensions of distance matrices are generated. Each instance is optimised using MapPoint and the corresponding optimal distance matrix, called the *TSP matrix* of this instance, is obtained from the pre-generated distance matrix. The *TSP solution* comprises *TSP tour* and *TSP tour length*, which represent the tour of the optimised instance and its total distance, respectively. The cities in the TSP tour is renumbered so the optimal tour becomes the identity permutation. As such, the TSP tour length is calculated from the optimal distance matrix

Table 1. Distance matrix.

Postcodes	1	2	3	4	5	6
1	0	8.63	8.42	1.65	6.13	5.21
2	8.63	0	2.63	9.96	11.34	3.26
3	8.42	2.63	0	9.51	17.46	7.71
4	1.65	9.96	9.51	0	5.23	6.54
5	6.13	11.34	17.46	5.23	0	9.82
6	5.21	3.26	7.71	6.54	9.82	0

Table 2. Optimal distance matrix.

Postcodes	1	2	3	4	5	6
1	0	17.46	9.51	8.42	7.71	2.63
2	17.46	0	5.23	6.13	9.82	11.34
3	9.51	5.23	0	1.65	6.54	9.96
4	8.42	6.13	1.65	0	5.21	8.63
5	7.71	9.82	6.54	5.21	0	3.26
6	2.63	11.34	9.96	8.63	3.26	0

using Equation (5) as follows:

$$\text{TSP tour length} = \sum_{i=1}^{n-1} c_{i,i+1} + c_{n,1} \quad (5)$$

Observe that such renumbering will not change the length of the optimum TSP tour.

By way of illustration, suppose a randomly generated instance of dimension, $n = 6$ postcodes, numbered as 1–6, and the corresponding *distance matrix* given in Table 1.

Now suppose this instance is optimised using MapPoint and the resulting *TSP tour* is given by $\langle 3,5,4,1,6,2 \rangle$. This TSP tour is then renumbered according to identity permutation $\langle 1,2,3,4,5,6 \rangle$, where Postcode 3 becomes Postcode 1, Postcode 5 becomes Postcode 2 and so on. The corresponding *optimal distance matrix*, known as *TSP matrix*, is shown in Table 2.

Using Equation (5), the total distance for this *TSP matrix* is 35.44 and this value is also referred to as the *TSP tour length* in further analysis. Subsequently, the TSP matrix is analysed for its robustness using the recognition of special structured matrices of Kalmanson. Three procedures based on combinatorial and linear programming (LP) approaches are proposed. For each procedure, the lengths, or total distances for the optimal tours of the Kalmanson matrices are computed using Equation (5). These optimal tours are the master tours which are robust to changes as shown by the theorems proposed by Deineko et al. (1995).

3.1. Combinatorial-based procedure (COMB-K)

Recognition of the Kalmanson matrix requires $O(n^2)$ time (Deineko et al., 1995). Due to the symmetricity property, we are only required to check the Kalmanson conditions in each row as well as on each element in the

upper diagonal of the distance matrix. If the symmetric TSP matrix does not hold the Kalmanson conditions, then some 'reductions' in the elements of the matrix are necessary to transform this matrix into a Kalmanson matrix. This can be accomplished by configuring Equations (3) and (4) stated earlier. Specifically, if any or both of these Equations is/ are violated, then the second term of each condition is proposed to be recalculated using the following Equations:

$$c_{i+1,j} = c_{i,j} + c_{i+1,j+1} - c_{i,j+1}$$

for all $1 \leq i \leq n-3, i+2 \leq j \leq n-1$ (6)

$$c_{i+1,n} = c_{i,n} + c_{i+1,1} - c_{i,1} \quad \text{for all } 2 \leq i \leq n-2 \quad (7)$$

3.2. Linear programming-based Kalmanson procedure

The procedure based on linear programming concerns a question which is how much distance should be 'reduced' from the elements of a *TSP matrix* such that the reduction is minimal and the resulting matrix sufficiently satisfies the Kalmanson conditions. This can be represented mathematically as shown in Equation (8):

$$a_{i,j} - \delta_{i,j} = c_{i,j}, \quad (8)$$

where $a_{i,j}$ is the element of the *TSP matrix*, $\delta_{i,j}$ is the distance to be reduced from $a_{i,j}$ and $c_{i,j}$ is the element of the *Kalmanson matrix*. This would be in contrast with the combinatorial-based procedure in which the latter deals with transforming the TSP matrix into the Kalmanson matrix regardless of the amount of reduction in each element.

The robustness analysis using LP-based procedures is implemented using Premium Solver Platform (PSP). PSP is an improved analysis tool developed by Frontline Systems Inc. which is able to solve a wide range of large-scale LP problems. A review of the software can be found in Albright (2001). In this method, two approaches are proposed and presented below.

3.2.1. Approach 1: LP₁

The first approach aims to minimise the maximum reduction value while fulfilling the Kalmanson conditions. The LP₁ formulation is as follows:

$$\text{Minimize } h \quad (9)$$

Subject to:

$$\delta_{i,j} \leq h \quad (10)$$

$$c_{i,j} = a_{i,j} - \delta_{i,j} \quad \text{for all } 1 \leq i \leq n; 1 \leq j \leq n \quad (11)$$

$$c_{i,j} + c_{i+1,j+1} - c_{i,j+1} - c_{i+1,j} \geq 0$$

for all $1 \leq i \leq n-3, i+2 \leq j \leq n-1$ (12)

$$c_{i,n} + c_{i+1,1} - c_{i,1} - c_{i+1,n} \geq 0 \quad \text{for all } 2 \leq i \leq n-2 \quad (13)$$

$$\delta_{i,j} \geq 0 \quad (14)$$

$$h \geq 0 \quad (15)$$

The objective function expressed in (9) requires the minimisation of the maximum reduction value, h . Constraint (10) ensures that each reduction value is at most h . It is imperative to note that $\delta_{i,j}$ is adjusted according to constraint (11) such that the reduced TSP matrix satisfies the Kalmanson conditions, denoted by constraints (12) and (13). Observe that these conditions are rearranged from inequality (3) and (4) which are the revised Kalmanson conditions. These constraints guarantee that the resulted distance matrix is a Kalmanson matrix. Constraints (14) and (15) specify the non-negativity value of $\delta_{i,j}$ and h . It is worth mentioning here that if the maximum reduction value is close to zero, then the TSP matrix is close to being a Kalmanson matrix. Subsequently, if the TSP matrix holds the special structures, then one can guarantee that the TSP tour is indeed a master tour and robust to changes.

Nevertheless, focusing on minimising the maximum reduction value as in (9) may result in a shorter total distance of the Kalmanson matrix. In fact, a preliminary analysis on arbitrary data showed that as the matrix dimension increases, the total distance of the Kalmanson matrix would be inclined towards the negative value. Hence, we perform a second analysis of the LP-based procedure with the objective of maximising the total distance of Kalmanson matrix. The linear programming formulation is stated below.

3.2.2. Approach 2: LP₂

$$\text{Maximize } \sum_{i=1}^{n-1} c_{i,i+1} + c_{n,1} \quad (16)$$

Subject to:

$$c_{i,j} = a_{i,j} - \delta_{i,j} \quad \text{for all } 1 \leq i \leq n; 1 \leq j \leq n \quad (17)$$

$$c_{i,j} + c_{i+1,j+1} - c_{i,j+1} - c_{i+1,j} \geq 0$$

for all $1 \leq i \leq n-3, i+2 \leq j \leq n-1$ (18)

$$c_{i,n} + c_{i+1,1} - c_{i,1} - c_{i+1,n} \geq 0 \quad \text{for all } 2 \leq i \leq n-2 \quad (19)$$

$$\delta_{i,j} \geq 0 \quad \text{for all } 1 \leq i \leq n; 1 \leq j \leq n \quad (20)$$

The objective of this approach as expressed in (16) is to maximise the total distance of the Kalmanson matrix.

This special structured matrix is obtained by reducing some elements of the TSP matrix by $\delta_{i,j}$, as imposed by constraint (17). Constraints (18) and (19) ensure that the resulting distance matrix satisfies the Kalmanson conditions. Also, the non-negativity value of $\delta_{i,j}$ is imposed in constraint (20).

The robustness procedures described above involve the reduction of elements of a *TSP matrix* in order to generate the special structured matrices. Thus, the total distances of Kalmanson matrices represent the lower bounds of the TSP tour length. To be specific, the three procedures implemented in this study produce three lower bounds. Obviously, the quality of these bounds is guaranteed as the special structured matrices possess a master tour. In order to investigate the significant difference of the *TSP tour length* from the lower bound, we measure the percentage deviation between the two using the formula:

$$\%dev = \frac{(\tau_{TSP} - \tau_{LB})}{\tau_{LB}} \times 100 \quad (21)$$

where τ_{TSP} and τ_{LB} are the total distances of the *TSP matrix* and the special structured matrices of Kalmanson. However, in cases where the lower bounds are negative, the denominator is changed to the minimum distance between two points in order to avoid negative deviations. Specifically, we have:

$$\%dev = \frac{(\tau_{TSP} - \tau_{LB})}{\min d_{i,j}} \times 100 \quad (22)$$

In addition, we also examined the reduction values in each instance across all dimensions and procedures. The reduction value is the value to be deducted from the element of the TSP matrix to transform it into the Kalmanson matrix. While this clarifies the value sufficient to obtain a master tour from an arbitrary matrix, the analysis also assists in observing whether the reduction values influence the performance of the lower bounds.

4. Computational results

The algorithms have been coded in the VBA programming language (Microsoft Visual Basic 6.5) with a direct link to Microsoft® MapPoint 2009 and Microsoft® Excel Premium Solver Platform and executed on a personal computer with an Intel (R) Core 2 Duo 3.0 GHz processor with 3.21 GB RAM.

Figure 2 illustrates the performance of the computed lower bounds and the TSP across all problem sizes while Figure 3 demonstrates the average deviations of the TSP from the lower bounds. In addition, Figure 4 depicts the average deviations of the TSP from the lower bounds obtained from the LP₂ approach. Also, Table 3 illustrates

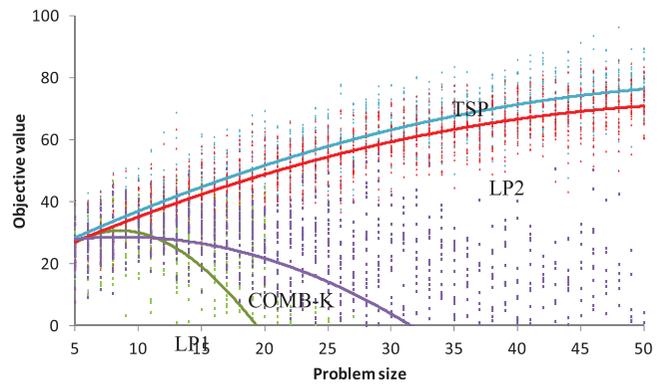


Figure 2. Illustration of TSP (optimal total distance of the *TSP matrix* from MapPoint) and the three lower bounds, namely COMB-K (total distance of the Kalmanson matrix from the combinatorial-based procedure), LP₁ (total distance of the Kalmanson matrix from LP₁) and LP₂ (total distance of the Kalmanson matrix from LP₂) on instances with dimensions of 5–50 (30 instances each).

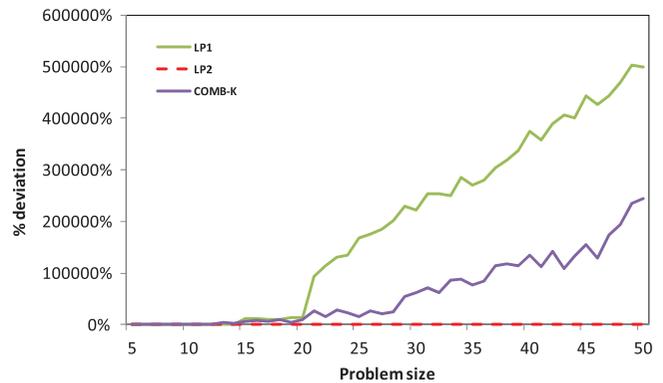


Figure 3. Average deviation of the TSP from the lower bounds for instances with dimensions of 5–50.

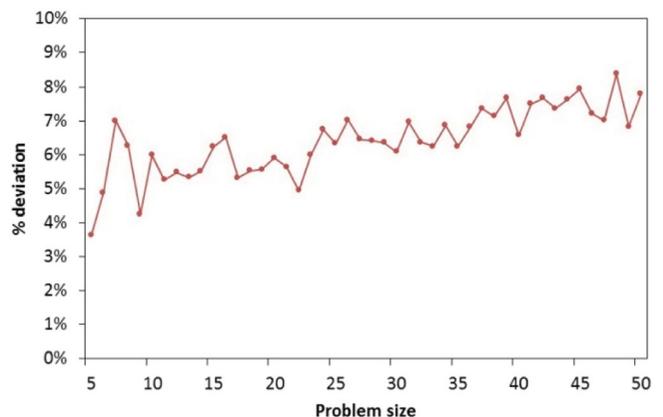


Figure 4. Average deviation of the TSP from the lower bounds of LP₂ for instances with dimensions of 5–50.

the performance of the three lower bounds compared to TSP for small dimension instances where median values are considered.

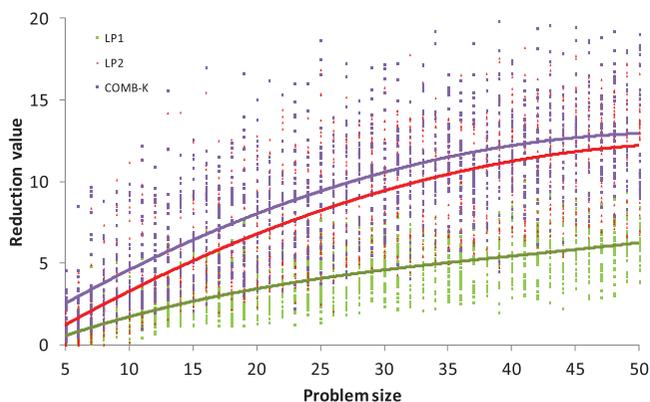
Table 3. Performance of lower bounds for small dimension instances.

Dimension	TSP	LP1	LP2	COMB_K
5	24.261	22.101	23.646*	23.044
6	27.949	26.209	26.933*	25.817
7	31.681	26.967	29.142*	27.517
8	28.847	24.442	27.455*	26.122
9	38.739	30.186	37.049*	33.843
10	38.482	29.483	35.198*	32.693
11	39.376	29.792	37.428*	30.556
12	39.407	29.248	37.633*	31.914
13	41.688	27.018	39.771*	27.831

*Indicates the best lower bound for the TSP.

It can be seen in Figure 2 that the *TSP tour length* increases as dimension gets larger. The lower bounds of COMB-K, LP₁, and LP₂ are relatively close for instances with small dimensions. In the cases when the dimension increases, it is observed that LP₂ clearly outperforms the other two procedures, where the total distances appear to be the closest to the TSP. On the contrary, it appears that the performance of COMB-K and LP₁ deteriorate much from the TSP as the matrix size increases. Indeed, the lower bounds of these procedures are getting worse as the values tend to drop to negative. This is evident by the large average deviations of the TSP from the two lower bounds as shown in Figure 3. This finding is supported by the nearly constant rate of the average deviations in Figure 4 that is about 6.4% from the LP₂, as well as the optimal lower bounds in Table 3. It is also worth noting that 30% of 5 × 5 instances have robust solutions according to the linear programming approach (LP₁ and LP₂).

In order to investigate the amount of reduction required for a random TSP matrix to adequately fulfil the Kalmanson conditions, we plot the reduction values of LP₁, LP₂, and COMB-K for instances across all problem sizes as depicted in Figure 5.

**Figure 5.** Reduction values of LP₁, LP₂ and COMB-K for instances with dimensions of 5–50.

The plot in Figure 5 allows one to examine the difference in the amount of reduction needed for a TSP matrix to sufficiently satisfy the special structured matrices produced by the three procedures. In almost all instances, the reduction values increase as the dimension becomes larger. Figure 5 shows that the average reduction values of LP₁ is the lowest and the best as approximately 4 miles on average is sufficient for a TSP matrix to possess a master tour. We can also see that most of the plots of LP₂ and COMB-K are overlapped, suggesting that there are similar reduction values between the two. Indeed, the average reduction values between these two procedures are relatively close to each other, which are 8.11 and 9.23 for LP₂ and COMB-K, respectively. Observing the reduction values within each dimension reveals no obvious trends. The fluctuation of values occurs possibly due to the randomness in the distribution of data.

Having compared the performance of the lower bounds and reduction values, it is also of interest to investigate whether the former is significantly influenced by the latter. It should be borne in mind that the approach of LP₁ is concerned with minimising the maximum reduction value while the LP₂ approach focuses on maximising the lower bounds. To reiterate, the analysis above indicates that LP₂ offers the best lower bounds, however the reduction value associated with LP₂ is not the lowest. In fact, as expected, whilst offering the smallest reduction values, LP₁ produces worse lower bounds in comparison with LP₂. Such observation indicates that the reduction value may not directly influence the lower bound, in the sense that it changes some of the distance matrix elements, but does not affect much the diagonal elements which constitute the optimal tour of the TSP instance.

5. Conclusions

In real-life problems, recognising special structures could assist the decision makers to design an optimal routing plan. If the designed tour is a master tour, then the optimal tour for a new problem can be obtained by simply skipping the points that are removed from the initial problem. In meal delivery problem, for example, the number of customers requesting daily services may be different as some customers may leave the service after some time. Thus, instead of rescheduling the customers every day which is obviously costly and time-consuming, designing a master tour will guarantee the robustness and optimality of a tour. In fact, we have presented that the concept of the master tour is closely related to Kalmanson matrices. Through the three procedures performed based on the special structures of Kalmanson matrices, namely COMB-K, LP₁ and LP₂, the results demonstrated that the proposed LP₂ approach performed consistently well and

could provide good lower bounds to the TSP tour lengths overall problem dimensions. Such observations indicated that LP_2 was considered as the best approximation to real life TSP for any number of dimensions. The percentage of the average deviation showed that a TSP tour length was within 7% from the lower bound generated by LP_2 . Nevertheless, LP_2 had slightly higher reduction values than LP_1 . The finding depicted that the Kalmanson matrix of LP_1 was relatively close to the TSP matrix where the difference in distance was around 4 miles on average, while LP_2 was twice the value.

The robustness analysis conducted in this study offers theoretical insight into the literature of routing-related problems. To the best of our knowledge, no single study that assesses the robustness of a routing plan based on the special structures of Kalmanson matrices has been published thus far. The recognition algorithms introduced in this paper are novel as they were developed and empirically tested on generated data for the first time. This study made a contribution by proposing and exploiting such theoretical concepts using implementation of combinatorial and linear programming algorithms in VBA programming language. Through iterative computational experiments and comparative analysis amongst procedures, this study has successfully demonstrated the applicability of the theoretical concepts to practical routing problems, identified the best lower bounds and reduction values, as well as established a generalisation of the findings on how far a TSP tour length is from a tour length of the Kalmanson matrix. Apparently, observing the bounds on the quality of the solution provided a theoretical understanding of the problems under study.

The proposed recognition algorithms which provide three lower bounds in this study could be applied to a wide range of test problems. For the combinatorial-based procedure, although no limitation on the problem size is imposed, there is a tendency to obtain worse lower bounds if the dataset used is large. On the contrary, the LP-based procedure could provide good lower bounds, however, the application may be restricted up to a certain problem size and this depends on the capability of the software used.

This study applied a generated data based on a real-life problem for the analysis. It is recommended that further experimentation of robustness analysis be undertaken on either a raw dataset or the outcomes of scheduling and routing exercise. For the former, the methods presented in this paper could assist in designing the robust routes and possibly identify further characteristics of these routes. This exercise could be done on any benchmark instances which could be easily obtained from various resources such as the TSP library (TSPLIB), VRP library (VRPLIB) or OR library (ORLIB), which then

allows comparability with previous best-known solutions. For the analysis of the solutions of existing studies, one could then examine the quality and robustness of the solutions obtained through the execution of recognition algorithms. Future research may also consider a multi-objective approach, for example, minimising the maximum reduction value as in LP_1 and maximising the total distance of Kalmanson matrix as in LP_2 . The results obtained from the robustness procedures could provide a recommendation to the decision makers on which routes to be implemented. The exercise will allow the decision makers to comprehend the underlying characteristics of the routes and possibly make the necessary modifications to ensure the optimality and robustness of the solutions regardless of any variations in the inputs. As such, these modifications could be done by considering the geographical regions of these services as specified in the current practice.

Disclosure statement

No potential conflict of interest was reported by the authors.

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References

- Albright, C. (2001). Software review: Premium solver platform for excel. *OR MS TODAY*, 28(3), 58–63.
- Amar, M. A., Khaznaji, W., & Bellalouna, M. (2017). An exact resolution for the probabilistic traveling salesman problem under the a priori strategy. *Procedia Computer Science*, 108, 1414–1423.
- Aziz, A., Quah, C., & Mohammad, N. (2016). Analyzing the efficiency of meals delivery transportation. *International Journal of Research in Business and Technology*, 9(1), 982–986.
- Baki, M. F., & Kabadi, S. N. (1998). A generalization of the convex-hull-and-line traveling salesman problem. *Journal of Applied Mathematics and Decision Sciences*, 2(2), 177–191.
- Balaprakash, P., Birattari, M., Stützle, T., & Dorigo, M. (2007). *An experimental study of estimation-based metaheuristics for the probabilistic traveling salesman problem* (Technical Report No. TR/IRIDIA/2007-021). Belgium: Université Libre de Bruxelles.
- Balaprakash, P., Birattari, M., Stützle, T., Yuan, Z., & Dorigo, M. (2009). Estimation-based ant colony optimization and local search for the probabilistic traveling salesman problem. *Swarm Intelligence*, 3(3), 223–242.
- Bartholdi, J. J., & Platzman, L. K. (1982). An $O(N \log N)$ planar travelling salesman heuristic based on spacefilling curves. *Operations Research Letters*, 1(4), 121–125.
- Berman, O., & Simchi-Levi, D. (1988). Finding the optimal a priori tour and location of a traveling salesman with nonhomogeneous customers. *Transportation Science*, 22(2), 148–154.
- Bertsimas, D., & Howell, L. H. (1993). Further results on the probabilistic traveling salesman problem. *European Journal of Operational Research*, 65(1), 68–95.
- Bianchi, L., & Dorigo, M. (2006). *Ant colony optimization and local search for the probabilistic traveling salesman problem: A case study in stochastic combinatorial optimization* (Doctoral Thesis). Université Libre de Bruxelles, Belgium.
- Bianchi, L., & Gambardella, L. M. (2007). *Ant colony optimization and local search based on exact and estimated objective values for the probabilistic traveling salesman problem* (Technical Report No. IDSIA-06-07). Manno: Dalle Molle Institute for Artificial Intelligence.
- Bianchi, L., Gambardella, L. M., & Dorigo, M. (2002). Solving the homogeneous probabilistic traveling salesman problem by the ACO metaheuristic. *Ant Algorithms*, 2463, 176–187.
- Bianchi, L., Knowles, J., & Bowler, N. (2005). Local search for the probabilistic traveling salesman problem: Correction to the 2-p-opt and 1-shift algorithms. *European Journal of Operational Research*, 162(1), 206–219.
- Birattari, M., Balaprakash, P., & Dorigo, M. (2005). *ACO/F-Race: Ant colony optimization and racing techniques for combinatorial optimization under uncertainty*. Paper read at MIC 2005: The 6th Metaheuristics International Conference.
- Birattari, M., Balaprakash, P., Stützle, T., & Dorigo, M. (2008). Estimation-based local search for stochastic combinatorial optimization using delta evaluations: A case study on the probabilistic traveling salesman problem. *INFORMS Journal on Computing*, 20(4), 644–658.
- Bowler, N. E., Fink, T. M. A., & Ball, R. C. (2003). Characterization of the probabilistic traveling salesman problem. *Physical Review E*, 68(3), 036703.
- Branke, J., & Guntsch, M. (2004). Solving the probabilistic TSP with ant colony optimization. *Journal of Mathematical Modelling and Algorithms*, 3(4), 403–425.
- Burkard, R. E., Deineko, V. G., van Dal, R., van der Veen, J. A. A., & Woeginger, G. J. (1998). Well-solvable special cases of the traveling salesman problem: A survey. *SIAM Review*, 40(3), 496–546.
- Castro, M., Sörensen, K., Vansteenwegen, P., & Goos, P. (2013). A memetic algorithm for the travelling salesperson problem with hotel selection. *Computers & Operations Research*, 40(7), 1716–1728.
- Christopher, G., Farach, M., & Trick, M. (1996). The structure of circular decomposable metrics. *Algorithms—ESA'96*, 1136, 486–500.
- Deineko, V., Rudolf, G. R., & Woeginger, G. J. (1995). *Sometimes travelling is easy: The master tour problem*. Paper read at European Symposium on algorithms.
- Ding, C., Cheng, Y., & He, M. (2007). Two-level genetic algorithm for clustered traveling salesman problem with application in large-scale TSPs. *Tsinghua Science & Technology*, 12(4), 459–465.
- Eldem, H., & Ülker, E. (2017). The application of ant colony optimization in the solution of 3D traveling salesman problem on a sphere. *Engineering Science and Technology, an International Journal*, 20(4), 1242–1248.
- Ezugwu, A. E., Adewumi, A. O., & Frincu, M. E. (2017). Simulated annealing based symbiotic organisms search optimization algorithm for traveling salesman problem. *Expert Systems with Applications*, 77(C), 189–210.
- Flood, M. M. (1956). The traveling-salesman problem. *Operations Research*, 4(1), 61–75.
- Gilmore, P. C., Lawler, E., Shmoys, D. B., & Lawler, E. (1985). The traveling salesman problem: A guided tour of combinatorial optimization. In E. L. Lawler, J. K. Lenstra, A. H. G. Rinnoy Kan, & D. B. Shmoys (Eds.), *Bulletin of the London mathematical society* (pp. 87–143). London: John Wiley & Sons.
- Gorodezky, I., Kleinberg, R. D., Shmoys, D. B., & Spencer, G. (2010). *Improved lower bounds for the universal and a priori TSP*. Paper read at APPROX-RANDOM.
- Gutin, G., & Punnen, A. P. (2006). *The Traveling Salesman problem and its variations* (Vol. 12). New York: Springer Science & Business Media.

- Hahn, H. (1932). Ergebnisse eines mathematischen Kolloquiums. *Monatshefte für Mathematik und Physik*, 39(1), A8–A8.
- Hajiaghayi, M. T., Kleinberg, R., & Leighton, T. (2006). *Improved lower and upper bounds for universal tsp in planar metrics*. Paper read at Proceedings of the Seventeenth Annual ACM-SIAM Symposium on Discrete algorithm.
- Hui, W. (2012). Comparison of several intelligent algorithms for solving TSP problem in industrial engineering. *Systems Engineering Procedia*, 4(C), 226–235.
- Jaillet, P. (1985). *Probabilistic traveling salesman problems* (Doctoral Thesis). Massachusetts Institute of Technology.
- Kalmanson, K. (1975). Edgeconvex circuits and the traveling salesman problem. *Canadian Journal of Mathematics*, 27(5), 1000–1010.
- Karaođlan, I., & Kesen, S. E. (2017). The coordinated production and transportation scheduling problem with a time-sensitive product: A branch-and-cut algorithm. *International Journal of Production Research*, 55(2), 536–557.
- Kóczy, L. T., Földesi, P., & Tű-Szabó, B. (2018). Enhanced discrete bacterial memetic evolutionary algorithm - An efficacious metaheuristic for the traveling salesman optimization. *Information Sciences*, 460-461(C), 389–400.
- Laporte, G., Louveaux, F. V., & Mercure, H. (1994). A priori optimization of the probabilistic traveling salesman problem. *Operations Research*, 42(3), 543–549.
- Lin, Y., Bian, Z., & Liu, X. (2016). Developing a dynamic neighborhood structure for an adaptive hybrid simulated annealing – tabu search algorithm to solve the symmetrical traveling salesman problem. *Applied Soft Computing*, 49(C), 937–952.
- Liu, Y.-H. (2007). A hybrid scatter search for the probabilistic traveling salesman problem. *Computers & Operations Research*, 34(10), 2949–2963.
- Liu, Y.-H. (2008). Solving the probabilistic travelling salesman problem based on genetic algorithm with queen selection scheme. In F. Greco (Ed.), *Traveling Salesman problem* (pp. 157–172). Vienna: InTech.
- Liu, Y.-H. (2010). Different initial solution generators in genetic algorithms for solving the probabilistic traveling salesman problem. *Applied Mathematics and Computation*, 216(1), 125–137.
- Liu, F., & Zeng, G. (2009). Study of genetic algorithm with reinforcement learning to solve the TSP. *Expert Systems with Applications*, 36(3, Part 2), 6995–7001.
- Lu, Y., Benlic, U., & Wu, Q. (2018). A hybrid dynamic programming and memetic algorithm to the traveling salesman problem with hotel selection. *Computers & Operations Research*, 90(C), 193–207.
- Mahfoudh, S. S., Khaznaji, W., & Bellalouna, M. (2015). A branch and bound algorithm for the probabilistic traveling salesman problem. In *Software Engineering, artificial intelligence, networking and parallel/distributed computing (SNPD), 2015 16th IEEE/ACIS International Conference on* (pp. 1–6). IEEE.
- Maity, S., Roy, A., & Maiti, M. (2017). An intelligent hybrid algorithm for 4-dimensional TSP. *Journal of Industrial Information Integration*, 5(C), 39–50.
- Meer, K. (2007). Simulated annealing versus metropolis for a TSP instance. *Information Processing Letters*, 104(6), 216–219.
- Papadimitriou, C. H. (2003). *Computational complexity*. Chichester: John Wiley and Sons.
- Rossi, F. A., & Gavioli, I. (1987). Aspects of heuristic methods in the probabilistic traveling salesman problem. *Advanced School on Stochastics in Combinatorial Optimization*, 214–227.
- Schalekamp, F. (2007). *Some results in universal and a priori optimization* (Doctoral Dissertation). Cornell University, Ithaca.
- Schalekamp, F., & Shmoys, D. B. (2008). Algorithms for the universal and a priori TSP. *Operations Research Letters*, 36(1), 1–3.
- Smith, M. M., & Chen, Y. S. (2016). A novel evolutionary algorithm for the homogeneous probabilistic Traveling Salesman problem. In *Computer and Information science (ICIS), 2016 IEEE/ACIS 15th International Conference on* (pp. 1–6). Okayama: IEEE.
- Wang, Y., Chen, Y., & Lin, Y. (2017). Memetic algorithm based on sequential variable neighborhood descent for the min-max multiple traveling salesman problem. *Computers & Industrial Engineering*, 106(C), 105–122.
- Weyland, D., Montemanni, R., & Gambardella, L. M. (2013). Heuristics for the probabilistic traveling salesman problem with deadlines based on quasi-parallel Monte Carlo sampling. *Computers & Operations Research*, 40(7), 1661–1670.
- Woeginger, G. J. (2002). *Computational problems that can be solved without computation*. Paper read at operations research Proceedings 2001.
- Zhou, Y., He, F., Hou, N., & Qiu, Y. (2018). Parallel ant colony optimization on multi-core SIMD CPUs. *Future Generation Computer Systems*, 79(Part 2), 473–487.