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An extended VIKOR method using stochastic data and subjective judgments

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A B S T R A C T

Decision makers (DMs) face different levels of uncertainty throughout the decision making process. In particular, natural language is generally subjective or ambiguous when used to express perceptions and judgments. The aim of this paper is to extend the VIKOR method and develop a methodology for solving multi-criteria decision making (MCDM) problems with stochastic data. The weights of the stochastic decision criteria considered in our extended VIKOR model have been determined using the fuzzy analytic hierarchy process (AHP) method. We present a case study in the banking industry to demonstrate the applicability of the proposed method. We also compare our results with the results obtained from a stochastic version of the super-efficiency data envelopment analysis (DEA) model to exhibit the efficacy of the procedures and algorithms.

1. Introduction

Multi-criteria decision making (MCDM) refers to making preference decisions (e.g., evaluation, prioritization and selection) over a set of available alternatives that are characterized by multiple and often conflicting criteria. Moreover, since decision making generally requires multiple perspectives from different people, most organizational decisions are made in groups (Ma, Lu, & Zhang, 2010). MCDM is used to select the most desirable alternative(s) from a set of available alternatives based on the selection criteria defined (Ju & Wang, 2013).

The classical MCDM frameworks assume that the ratings and the weights of the criteria are known precisely. However, many real-world problems involve uncertain data and one cannot assume the knowledge and judgments of the decision makers (DMs) or experts to be precise (Sayadi, Heydari, & Shahanaghi, 2009). MCDM models account for different types of uncertainties, which are generally modeled using stochastic analysis or fuzzy set theory. The stochastic approach is more suitable when a probabilistic data set represents the existing uncertainty, while the fuzzy approach is more appropriate when the parameters are vague and ambiguous (Zarghami & Szidarovszky, 2009).

The VIKOR method was introduced by Opricovic in 1998 to model the multi-criteria optimization of complex systems (Opricovic, 1998). This method focuses on ranking and selecting from a set of available alternatives in the presence of conflicting criteria by proposing a compromise solution (composed by either one or several alternatives) (Opricovic & Tzeng, 2007). A compromise solution is often preferred to an optimal solution because selection criteria are usually in conflict. The best alternative is chosen to be the one with the smallest distance to the positive ideal solution using a particular measure of "closeness".

The VIKOR method is suitable for those situations where the goal is to maximize profit while the risk of the decisions is deemed to be less important. The major advantage of the VIKOR method is that it can trade off the maximum group utility of the "majority" and the minimum individual regret of the "opponent". In addition, the required calculations are simple and straightforward (Bazzazi, Osanloo, & Karimi, 2011).

In this paper, we extend the basic structure of VIKOR and develop a methodology for solving MCDM problems where the data describing the performance of the alternatives are stochastic. The proposed model considers multiple stochastic criteria, whose

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weights have been determined applying the fuzzy analytic hierarchy process (AHP) method to the linguistic judgments provided by different experts. We present a case study in the banking industry to demonstrate the applicability of the proposed method. Moreover, we compare our results with the results obtained from a stochastic version of the super-efficiency data envelopment analysis (DEA) model in order to exhibit the efficacy of the procedures and algorithms.

The remainder of this paper is organized as follows. The next section provides a short review of the VIKOR literature. A brief introduction to the VIKOR method is presented in Section 3. In Section 4, the extended stochastic VIKOR method proposed is described. Section 5 provides an illustrative example to show the applicability of the extended VIKOR method. Section 6 compares the ranking obtained using our model with the one derived from applying the super-efficiency stochastic DEA model. Section 7 concludes and suggests future research directions.

2. Literature review and contribution

The VIKOR method has been extensively applied to solve different types of MCDM problems both in certain settings and in fuzzy environments with subjective judgments.

Within the former settings, Chang and Hsu (2009) used VIKOR to prioritize land-use restraint strategies in the Tseng–Wen reservoir watershed. Sayadi et al. (2009) extended the VIKOR method in order to solve decision making problems with interval numbers. Liou, Tsai, Lin, and Tzeng (2010) used a modified VIKOR method for improving the service quality of domestic airlines. Chatterjee, Athawale, and Chakraborty (2009) applied the VIKOR procedure to the selection process of materials for flywheel and sailing boat mast design. These authors obtained a complete ranking of the materials by considering many criteria related to the actual applications of the respective products. In this regard, Civic and Vucijak (2014) considered several selected criteria to evaluate insulation options that increase energy efficiency in buildings and applied the VIKOR method to rank the options and select the best one.

The use of fuzzy sets gives DMs enough flexibility to incorporate unquantifiable, incomplete and partial information into a decision model (Chou, Hsu, & Chen, 2008). Fuzzy MCDM, with the capacity to resolve the lack of precision in measuring the importance weights of the criteria and the corresponding ratings of alternatives, has been widely applied to address decision making problems with multiple criteria and alternatives in a consistent way. For instance, Chang (2014) proposed a framework based on several concepts from fuzzy set theory and the VIKOR method to provide a systematic process for evaluating the quality of hospital services in a fuzzy environment.

Indeed, many researchers have incorporated elements from different fuzzy environments into their modified VIKOR models. For example, Chen and Wang (2009) optimized the choice of partners in IS/IT outsourcing projects following a fuzzy VIKOR approach. Sanayei, Mousavi, and Yazdankhah (2010) and Shemshadi, Shirazi, Toreihi, and Tarokh (2011) developed different fuzzy VIKOR methods for a supplier selection problem with linguistic ratings and weights. In addition, the latter authors used an entropy measure to assign the weights of the criteria. Recent applications of the fuzzy VIKOR method are quite varied and range from water resource planning (Opricovic, 2011) to the selection of robots for handling materials (Devi, 2011).

Finally, as we do in the current paper, the fuzzy VIKOR method has been integrated with other MCDM techniques to determine the ranking of alternatives. Kuo and Liang (2011) evaluated the service quality of airports using a MCDM technique that combined fuzzy VIKOR and grey relational analysis. Kaya and Kahraman (2010) designed an integrated fuzzy VIKOR and AHP methodology for multi-criteria renewable energy planning in Istanbul. In the same way, Kaya and Kahraman (2011) integrated VIKOR and the AHP method to select alternative forestation areas. They determined the weights of the criteria using a fuzzy AHP approach in order to allow for both pairwise comparisons and the utilization of linguistic variables.

In this regard, our model considers a fuzzy scenario where linguistic expert evaluations are used to determine the weights of the decision criteria. Thus, similarly to the latter authors, the weights that follow from these evaluations have been computed using a fuzzy AHP approach. Then, these weights have been integrated within VIKOR to provide a ranking of the different alternatives. However, differently from the above models, ours assumes that the data available to measure the performance of the alternatives are stochastic. As a result, our model allows the DMs to integrate within VIKOR linguistic evaluations regarding the relative importance of the criteria used to classify alternatives whose performance is described by stochastic data.

3. The VIKOR method

The basic idea of the VIKOR technique, a MCDM method introduced by Opricovic (1998), consists of defining positive and negative ideal points to determine the relative distance of each alternative. After each relative distance is calculated, a weighted compromise ranking is obtained to determine the importance of the m alternatives available, x_j, with $j = 1, 2, \ldots, m$. VIKOR provides a particularly effective tool in MCDM situations where the DM is unable "to express his/her preference at the beginning of system design" (Opricovic & Tzeng, 2004, p. 448). The compromise-ranking algorithm is composed of the following steps:

1. Define the rating functions f_{ij}, which provide the value of the i-th criterion function for alternative x_j, with $i = 1, 2, \ldots, n$. Calculate the best, f_i^+, and the worst, f_i^-, values of all criterion functions. If the criterion being consider constitutes a benefit (i.e. it is a positive criterion), the corresponding values are defined as follows

$$f_i^+ = \max[(f_{ij})|j = 1, 2, \ldots, m] \tag{1}$$

$$f_i^- = \min[(f_{ij})|j = 1, 2, \ldots, m] \tag{2}$$

2. Compute the values S_j and R_j, $j = 1, 2, \ldots, m$, using the following relations

$$S_j = \sum_{i=1}^{n} w_i \frac{(f_i^+ - f_{ij})}{(f_i^+ - f_i^-)} \tag{3}$$

$$R_j = \max_i \left[w_i \frac{(f_i^+ - f_{ij})}{(f_i^+ - f_i^-)} \right] \tag{4}$$

where S_j and R_j represent the group utility measure and the individual regret measure defined for each alternative x_j, respectively, and w_i are the weights of the criteria that reflect their relative importance.

3. Compute the values Q_j, $j = 1, 2, \ldots, m$, using the relation

$$Q_j = v\left[\frac{(S_j - S^+)}{(S^- - S^+)}\right] + (1 - v)\left[\frac{(R_j - R^+)}{(R^- - R^+)}\right] \tag{5}$$

where

$$S^+ = Min[(S_j)|j = 1, 2, \ldots, m] \tag{6}$$

$$S^- = Max[(S_j)|j = 1, 2, \ldots, m] \tag{7}$$

$$R^{+} = Min[(R_j)|j = 1, 2, \ldots, m] \quad (8)$$

$$R^{-} = Max[(R_j)|j = 1, 2, \ldots, m] \quad (9)$$

and v is the weight introduced to support the strategy of maximum group utility while $(1 - v)$ is used to weight the individual regret. A commonly assumed value for this parameter is $v = 0.5$ (Kackar, 1985).

4. Rank the alternatives, sorting by the values (S_j, R_j, Q_j). The results are three ranking lists that can be used to propose and validate a compromise solution (Opricovic & Tzeng, 2004).

The resulting compromise solution provides a balance between the maximum group utility of the "majority" (represented by $\min S_j$) and the minimum individual regret of the "opponent" (represented by $\min R_j$) (Sayadi et al., 2009).

It should be mentioned that, although the VIKOR method has numerous advantages, the performance ratings, f_{ij}, are generally quantified using crisp values. However, under many circumstances, such as those where DMs are unable to express their preferences accurately, crisp data are inadequate to model real-life situations. That is, given the fact that human judgments including preferences are often vague, it is both difficult and inaccurate to rate them as exact numerical values (Bazzazi et al., 2011).

4. Extended VIKOR method for decision making problems with stochastic data

Stochastic data constitute the simplest way of representing uncertainty in a decision matrix, which makes them natural candidates to deal with decision making problems in imprecise and uncertain environments. Specifying a stochastic interval domain for an entry parameter in a decision matrix indicates that the realizations of the entry parameter can take any value within the interval based on their associated probability density. In this paper, it is assumed (though it will be verified in the case study) that the data follow a normal distribution. In addition, when stochastic data are used to represent the extent of tolerance or the domain of the potential values that an entry parameter can take, the coefficient of variation can be used to measure the uncertainty associated with a set of realizations.

The coefficient of variation (cv) is a statistical measure of the distribution of realizations in a data series around the mean. It represents the ratio of the standard deviation to the mean. That is, given a sample of l realizations, y_k, $k = 1, \ldots, l$, the sample mean, $\bar{y} = \frac{\sum y_k}{l}$, and the standard deviation, $\sigma = \sqrt{\frac{1}{l}\sum_{k=1}^{l}(y_k - \bar{y})^2}$, the coefficient of variation is defined as $cv = \frac{\sigma}{\bar{y}}$.

In order to extend the VIKOR method to solve MCDM problems with stochastic data, suppose that a decision matrix with stochastic data has the following form:

	C_1	C_2	$\ldots$	C_n
A_1	$[\bar{f}_{11}, cv_{11}]$	$[\bar{f}_{21}, cv_{21}]$	$\ldots$	$[\bar{f}_{n1}, cv_{n1}]$
A_2	$[\bar{f}_{12}, cv_{12}]$	$[\bar{f}_{22}, cv_{22}]$	$\ldots$	$[\bar{f}_{n2}, cv_{n2}]$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
A_m	$[\bar{f}_{1m}, cv_{1m}]$	$[\bar{f}_{2m}, cv_{2m}]$	$\ldots$	$[\bar{f}_{nm}, cv_{nm}]$

$$W = [w_1, w_2, \ldots, w_n]$$

where $A_1, A_2, \ldots, A_m$ are the available alternatives among which DMs have to choose, $C_1, C_2, \ldots, C_n$ are the criteria used to measure the performance of the alternatives, and $w_i (i = 1, 2, \ldots, n)$ are the weights of the criteria, representing the relative importance assigned by the DM to each criterion.

The $\bar{f}_{ij}$ entries of the matrix correspond to the rating functions, which provide the value of the i-th mean criterion for the j-th alternative. The coefficient of variation associated to the i-th mean criterion and the j-th alternative is denoted by cv_{ij}. The proposed VIKOR method consists of the following steps:

1. Determine the best f_i^{+} and the worst f_i^{-} values of all rating functions both for positive (benefit) and negative (loss) criteria. The different approaches followed, based on the type of criterion being considered, are described in the equations below. Note that, in all cases, the spread used to determine the extremes of the rating functions is based on the maximum coefficient of variation obtained for each criterion, $\max_j cv_{ij}$.

f_i^{+}

$$\text{Positive criterion } f_i^{+} = \max \bar{f}_{ij} \times (1 + \max_j cv_{ij}), \quad \forall j: \quad j = 1, 2, \ldots, m \quad (10)$$

$$\text{Negative criterion } f_i^{+} = \min \bar{f}_{ij} \times (1 - \max_j cv_{ij}), \quad \forall j: \quad j = 1, 2, \ldots, m \quad (11)$$

f_i^{-}

$$\text{Positive criterion } f_i^{-} = \min \bar{f}_{ij} \times (1 - \max_j cv_{ij}), \quad \forall j: \quad j = 1, 2, \ldots, m \quad (12)$$

$$\text{Negative criterion } f_i^{-} = \max \bar{f}_{ij} \times (1 + \max_j cv_{ij}), \quad \forall j: \quad j = 1, 2, \ldots, m \quad (13)$$

2. Compute the corresponding values S_j, and R_j, $j = 1, 2, 3, \ldots, m$

Positive criterion S_j

$$= \sum_{i=1}^{n} w_i \frac{\max \bar{f}_{ij} \times (1 + \max_j cv_{ij}) - \bar{f}_{ij}}{\max \bar{f}_{ij} \times (1 + \max_j cv_{ij}) - \min \bar{f}_{ij} \times (1 - \max_j cv_{ij})} \quad (14)$$

Negative criterion S_j

$$= \sum_{i=1}^{n} w_i \frac{\min \bar{f}_{ij} \times (1 - \max_j cv_{ij}) - \bar{f}_{ij}}{\min \bar{f}_{ij} \times (1 - \max_j cv_{ij}) - \max \bar{f}_{ij} \times (1 + \max_j cv_{ij})} \quad (15)$$

Positive criterion R_j

$$= \max_i \left\{ w_i \frac{\max \bar{f}_{ij} \times (1 + \max_j cv_{ij}) - \bar{f}_{ij}}{\max \bar{f}_{ij} \times (1 + \max_j cv_{ij}) - \min \bar{f}_{ij} \times (1 - \max_j cv_{ij})} \right\} \quad (16)$$

Negative criterion R_j

$$= \max_i \left\{ w_i \frac{\min \bar{f}_{ij} \times (1 - \max_j cv_{ij}) - \bar{f}_{ij}}{\min \bar{f}_{ij} \times (1 - \max_j cv_{ij}) - \max \bar{f}_{ij} \times (1 + \max_j cv_{ij})} \right\} \quad (17)$$

These functions represent the group utility measure, S_j, and the individual regret measure, R_j, computed by the DM based on the variability exhibited by the stochastic observations of the different alternatives. Note that, when VIKOR is based on deterministic data, Eqs. (3) and (4) illustrate how each realization is directly compared to the limit values of the corresponding rating functions. However, in the current setting, we must also account for the variability inherent to the set of realizations observed, which is reflected in the respective coefficients of variation, when defining the limit values of the rating functions.

3. Calculate the values $Q_j, j = 1, 2, \ldots, m$, using the relation

$$Q_j = v\left[\frac{(S_j - S^+)}{(S^- - S^+)}\right] + (1 - v)\left[\frac{(R_j - R^+)}{(R^- - R^+)}\right] \tag{18}$$

Here, v is interpreted as a proxy for the optimism level of the DM ($0 < v \leqslant 1$). In particular, an optimistic DM can be assumed to assign a higher value to v than a pessimistic one. That is, an optimistic DM emphasizes the utility of the group while a pessimistic one focuses on individual regret. In this regard, it can be assumed that a neutral (neither optimistic nor pessimistic) DM defines $v = 0.5$. In this case, the ranking results obtained would be similar to those derived from comparing the stochastic means of each alternative.

4. Rank the alternatives, sorting by the values (S_j, R_j, Q_j), and propose a compromise solution.

5. Numerical example

In this section, a numerical example is presented to illustrate how the proposed method can be applied to evaluate the performance efficiency of bank branches.

5.1. Data acquisition, alternatives and criteria

In this case study, 22 branches of the Peoples Bank[1] in East Virginia have been selected $(A_1, A_2, \ldots, A_{22})$ and 7 criteria $(C_1, C_2, \ldots, C_7)$ chosen for their evaluation, namely, suspicious receivables cost (C_1), personnel cost (C_2), capital cost (C_3), branch equipment cost (C_4), incomes (C_5), deposits (C_6) and banking facilities (C_7). The first four criteria represent negative (input) variables while the last three correspond to positive (output) ones. The unit in which these criteria are expressed is million dollars. The data on the criteria for each one of the 22 branches were gathered over a period of 8 different weeks.

More precisely, the data have been retrieved from the financial statements of 22 bank branches located within an East Virginian county, which allows for a direct comparison among them through the period of analysis. The financial statements of each branch present a detailed description of its deposits, incomes and operational costs. In particular, these statements provide sufficiently disaggregated information so as to account for the different types of costs considered. Note that we have included suspicious receivables among the costs being considered. Suspicious receivables specify the debts owed to the bank, even if not currently due, that may not be repaid. Clearly, the costs suffered by the bank while trying to collect its receivables should be minimized. Moreover, the inability of a branch to identify defaulters among its borrowers may be considered as a proxy for suboptimal performance. On the positive criteria side, we have considered the capacity of the bank to provide facilities to local companies as a desirable criterion (output) together with incomes and deposits.

After collecting the data, we performed the Kolmogorov–Smirnov test using SPSS software in order to verify whether they followed a normal distribution. Table 1 presents the results of the test sorted per criterion and week.

Note that the Z and p values of the Kolmogorov–Smirnov test are not significant for any of the criteria entries, i.e. the null-hypothesis that our samples are drawn from a normal distribution holds.

5.2. Computing the stochastic decision matrix

After verifying the stochastic nature of the data acquired, we can assume that the numerical values of the criteria are imprecise and deal accordingly with the uncertainty inherent to the decision problem. As a result, we have calculated the coefficient of variation for each branch and criterion and illustrated the corresponding stochastic decision matrix in Tables 2 and 3.

5.3. VIKOR analysis with stochastic data

The weights assigned to each of the 7 criteria have been calculated using the extent analysis method on the fuzzy AHP methodology. In order to do so, a questionnaire was designed and the opinions of 7 banking experts regarding the relative importance of each criterion were obtained in linguistic terms. Then, the corresponding triangular fuzzy numbers were determined. In this regard, the 7 numbers in each parenthesis within Table 4 represent the opinions of experts, whose associated linguistic terms and triangular fuzzy scales can be found in Table 2 of Sen and Cinar (2010).

Finally, the weights of the criteria were computed following the methodology proposed in Sen and Cinar (2010), who apply the extent analysis method and the principle of triangular fuzzy number comparison to obtain the corresponding importance weights. The resulting criteria weights are given by:

$$W = (0.08671, 0.04667, 0.25633, 0.03724, 0.18773, 0.22411, 0.16121)$$

Note that criteria (C_3) and (C_6) have the highest weights, which implies that management should focus on capital cost and deposits as the main criteria promoting the performance of the bank. This can be done, for example, by optimally planning the distribution of its capital costs and by designing incentive programs for attracting more customers' deposits.

In order to implement the extended VIKOR method, the following steps should be taken.

1. Calculate f_i^+ and f_i^- using Eqs. (10)–(13). The resulting numerical values are shown in Tables 5 and 6.
2. Calculate S_j and R_j using Eqs. (14)–(17). Then, compute the values of Q_j using Eq. (18). For this purpose, the following values are derived from Table 2.

$$S^- = 0.7444 \quad S^+ = 0.1944$$
$$R^- = 0.2563 \quad R^+ = 0.0683$$

The three ranking lists obtained are presented in Table 7.

Finally, following step 4 of the method proposed in Section 4, the final ranking of the 22 branches based on the selected criteria is given by:

$$A_4 > A_{15} > A_7 > A_{18} > A_{11} > A_6 > A_{16} > A_{17} > A_{14} > A_{19} > A_{20}$$
$$> A_1 > A_{22} > A_5 > A_9 > A_8 > A_{10} > A_{21} > A_3 > A_{13} > A_{12} > A_2$$

It can be easily verified that the solution defined by alternative A_4 constitutes an acceptable compromise solution, since it satisfies the *Acceptable advantage* and the *Acceptable stability in decision making* conditions defined by Opricovic and Tzeng (2004). That is, $Q_{15} - Q_4 = 0.0641 > \frac{1}{22-1} = 0.0476$ and A_4 takes the minimum scores among the S_j and R_j values. Thus, based on the proposed extended VIKOR methodology, the 4th branch exhibits the best performance in this case study.

Consider now the next most preferred alternative. Note that, after A_4, the 15th branch has the second lowest value of S_j, while the 7th branch has the second lowest value of R_j. The ranking arising from the proposed methodology is therefore determined by the values of Q_j. Consequently, the A_{15} alternative has been assigned to the second place. We have explicitly described this result in order to highlight the fact that it is possible for A_7 to

[1] The name is changed to protect the anonymity of the bank.

Table 1
Normal distribution test of criteria.

Week	K–S	Criterion						
		C_1	C_2	C_3	C_4	C_5	C_6	C_7
1	Kolmogorov–Smirnov Z	0.533	0.410	0.667	0.768	0.595	0.602	0.445
	Asymp. Sig. (2-tailed)	0.939	0.996	0.766	0.597	0.871	0.862	0.989
2	Kolmogorov–Smirnov Z	0.524	0.498	0.549	0.687	0.682	0.537	0.380
	Asymp. Sig. (2-tailed)	0.947	0.965	0.924	0.733	0.741	0.935	0.999
3	Kolmogorov–Smirnov Z	0.638	0.446	0.591	0.678	0.596	0.618	0.407
	Asymp. Sig. (2-tailed)	0.810	0.989	0.876	0.748	0.870	0.840	0.996
4	Kolmogorov–Smirnov Z	0.615	0.273	0.974	0.606	0.698	0.522	0.388
	Asymp. Sig. (2-tailed)	0.843	1.000	0.299	0.856	0.715	0.948	0.998
5	Kolmogorov–Smirnov Z	0.550	0.441	0.852	0.678	0.522	0.605	0.372
	Asymp. Sig. (2-tailed)	0.923	0.990	0.463	0.748	0.948	0.857	0.999
6	Kolmogorov–Smirnov Z	0.705	0.436	0.637	0.637	0.592	0.697	0.460
	Asymp. Sig. (2-tailed)	0.703	0.991	0.812	0.811	0.874	0.717	0.984
7	Kolmogorov–Smirnov Z	0.501	0.574	0.648	0.687	0.672	0.593	0.435
	Asymp. Sig. (2-tailed)	0.964	0.897	0.795	0.732	0.758	0.873	0.991
8	Kolmogorov–Smirnov Z	0.627	0.486	0.532	0.625	0.624	0.622	0.573
	Asymp. Sig. (2-tailed)	0.827	0.972	0.939	0.830	0.830	0.834	0.898

Table 2
$(\bar{f}_{ij}, cv)$ of inputs.

Branch number	C_1		C_2		C_3		C_4	
	$\bar{X}$	cv	$\bar{X}$	cv	$\bar{X}$	cv	$\bar{X}$	cv
A_1	0.183092	6.5903×10^{-5}	0.212985	1.24011×10^{-5}	0.235626	0.000113243	0.184808	0.000162925
A_2	0.208596	7.08299×10^{-5}	0.230363	6.08144×10^{-5}	0.245014	7.14695×10^{-5}	0.241272	2.59317×10^{-5}
A_3	0.187978	0.000156629	0.252706	4.93194×10^{-6}	0.209421	1.76645×10^{-5}	0.171436	6.26938×10^{-5}
A_4	0.225785	3.56953×10^{-5}	0.166362	2.14093×10^{-5}	0.177091	6.60898×10^{-5}	0.234273	8.39646×10^{-5}
A_5	0.25391	2.98975×10^{-5}	0.20528	1.29488×10^{-5}	0.223603	6.24711×10^{-5}	0.249573	5.52285×10^{-5}
A_6	0.214122	5.98553×10^{-6}	0.177374	6.41937×10^{-5}	0.191373	0.000100643	0.164826	6.77854×10^{-5}
A_7	0.21015	3.18718×10^{-5}	0.239554	2.6188×10^{-5}	0.187291	9.16315×10^{-5}	0.189364	7.77015×10^{-5}
A_8	0.243531	0.000105899	0.221798	2.4126×10^{-5}	0.223191	8.7938×10^{-5}	0.234913	9.66622×10^{-5}
A_9	0.181421	6.17473×10^{-5}	0.187905	3.84803×10^{-5}	0.214526	3.64076×10^{-5}	0.210105	4.21462×10^{-5}
A_{10}	0.250902	1.82477×10^{-5}	0.165296	0.000146387	0.22845	8.40266×10^{-5}	0.198068	0.000154162
A_{11}	0.156557	3.98902×10^{-5}	0.260051	5.74954×10^{-5}	0.214174	9.94717×10^{-5}	0.25599	9.93986×10^{-6}
A_{12}	0.190552	3.79299×10^{-5}	0.196073	0.00011312	0.231224	9.193×10^{-5}	0.222687	7.58071×10^{-5}
A_{13}	0.253144	3.79491×10^{-5}	0.206956	8.06985×10^{-5}	0.224874	0.000136051	0.167009	8.48621×10^{-5}
A_{14}	0.243138	5.10019×10^{-5}	0.180417	0.000104778	0.192666	0.000162396	0.251582	7.6942×10^{-5}
A_{15}	0.202739	4.07969×10^{-5}	0.220127	4.42057×10^{-5}	0.186853	6.34488×10^{-5}	0.186592	3.07689×10^{-5}
A_{16}	0.238221	2.85497×10^{-5}	0.254391	4.22661×10^{-5}	0.228408	6.22422×10^{-5}	0.19576	7.77746×10^{-5}
A_{17}	0.192813	4.36255×10^{-5}	0.229055	0.000100119	0.175677	6.75783×10^{-5}	0.260848	2.45347×10^{-5}
A_{18}	0.224648	5.34604×10^{-5}	0.201761	7.98192×10^{-5}	0.17529	7.81931×10^{-5}	0.209711	6.92383×10^{-5}
A_{19}	0.195362	2.14551×10^{-5}	0.17204	5.52486×10^{-5}	0.213239	8.8118×10^{-5}	0.19405	8.61807×10^{-5}
A_{20}	0.178847	7.0373×10^{-5}	0.240755	6.96634×10^{-5}	0.208716	0.000161973	0.258876	9.15553×10^{-5}
A_{21}	0.23379	1.51622×10^{-5}	0.232072	2.87053×10^{-5}	0.251264	6.66647×10^{-5}	0.183353	5.70796×10^{-5}
A_{22}	0.180214	4.28247×10^{-5}	0.194373	6.75136×10^{-5}	0.226479	7.09521×10^{-5}	0.171603	6.16828×10^{-5}

overtake A_{15} in the ranking by improving its performance, particularly the one based on the criteria endowed with the highest importance weights.

Similarly, the rest of the neighboring alternatives compete to get a better position in the ranking. Consider, for example, A_2, which has a lower S_j but a higher R_j than A_{12}. However, this latter branch exhibits a better Q_j performance. In this regard, the final ranking obtained depends directly on the subjective value assigned by the DM to the variable v.

6. Stochastic data envelopment analysis

We now compare the ranking obtained using the stochastic data version of VIKOR introduced in this paper with a stochastic version of the super-efficiency DEA model introduced by Andersen and Petersen (1993). DEA is a non-parametric technique designed for evaluating the relative productive efficiency of decision making units (DMUs) that produce multiple-outputs using multiple-inputs. This problem was addressed by Charnes, Cooper, and Rhodes (1978), who introduced the first DEA model, generally denoted by CCR. This seminal model is deterministic in nature though several stochastic extensions were quickly developed in the literature (Cooper, Deng, Huang, & Li, 2004).

Following the seminal work of Andersen and Petersen (1993), the DEA literature has recently focused on addressing the question of how to rank efficient DMUs, giving rise to the super-efficiency branch of DEA. Though mainly deterministic,

Table 3
$(\bar{f}_{ij}, cv)$ of outputs.

Branch number	C_5		C_6		C_7	
	$\bar{X}$	cv	$\bar{X}$	cv	$\bar{X}$	cv
A_1	0.242661	1.98016×10^{-5}	0.17367	4.53261×10^{-5}	0.211735	1.34646×10^{-5}
A_2	0.152329	2.62357×10^{-5}	0.220824	9.33601×10^{-6}	0.174535	2.03572×10^{-5}
A_3	0.151927	1.46208×10^{-5}	0.166938	4.69472×10^{-5}	0.14435	3.64534×10^{-5}
A_4	0.229268	1.86922×10^{-5}	0.29375	2.51355×10^{-5}	0.247675	2.12795×10^{-5}
A_5	0.249618	2.05784×10^{-5}	0.205499	3.61937×10^{-5}	0.20643	2.6886×10^{-5}
A_6	0.184324	1.35987×10^{-5}	0.1945	3.31699×10^{-5}	0.271635	1.9659×10^{-5}
A_7	0.226834	2.49185×10^{-5}	0.251782	1.84688×10^{-5}	0.212217	1.49143×10^{-5}
A_8	0.175298	1.50756×10^{-5}	0.167357	3.30199×10^{-5}	0.254375	1.07385×10^{-5}
A_9	0.217959	1.4327×10^{-5}	0.15038	3.14595×10^{-5}	0.164785	3.47518×10^{-5}
A_{10}	0.19223	2.53435×10^{-5}	0.179498	4.64395×10^{-5}	0.160314	3.30879×10^{-5}
A_{11}	0.252283	1.67157×10^{-5}	0.250188	2.16852×10^{-5}	0.166095	3.99225×10^{-5}
A_{12}	0.157571	1.87832×10^{-5}	0.149209	4.06469×10^{-5}	0.186626	2.92354×10^{-5}
A_{13}	0.198878	4.26909×10^{-5}	0.164063	6.27114×10^{-5}	0.129739	2.42102×10^{-5}
A_{14}	0.177068	2.27906×10^{-5}	0.247196	1.66502×10^{-5}	0.221459	2.29682×10^{-5}
A_{15}	0.240081	1.78092×10^{-5}	0.28428	1.01969×10^{-5}	0.204324	1.97875×10^{-5}
A_{16}	0.264702	9.56793×10^{-6}	0.277888	1.91376×10^{-5}	0.228287	2.20661×10^{-5}
A_{17}	0.152925	2.63126×10^{-5}	0.223313	1.08212×10^{-5}	0.279775	9.429×10^{-6}
A_{18}	0.273498	5.52225×10^{-6}	0.193881	1.83026×10^{-5}	0.264884	2.8007×10^{-5}
A_{19}	0.264819	2.62788×10^{-5}	0.18166	4.41206×10^{-5}	0.189833	4.64215×10^{-5}
A_{20}	0.198868	3.64654×10^{-5}	0.217204	2.84405×10^{-5}	0.19879	1.45533×10^{-5}
A_{21}	0.241816	3.61768×10^{-6}	0.184588	8.06913×10^{-6}	0.232329	2.02833×10^{-5}
A_{22}	0.160726	3.17916×10^{-5}	0.218817	5.60012×10^{-5}	0.251616	3.02976×10^{-5}

Table 4
Expert opinions on the relative importance of criteria.

Criterion	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	$(\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1})$	$(\tilde{2},\tilde{2},\tilde{2},\tilde{3},\tilde{1},\tilde{2},\tilde{2})$	$(\widetilde{\frac{1}{2}},\widetilde{\frac{1}{5}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{5}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}})$	$(\tilde{2},\tilde{2},\tilde{1},\tilde{2},\tilde{2},\tilde{2},\tilde{2})$	$(\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{3}})$	$(\widetilde{\frac{1}{3}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{5}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}})$	$(\widetilde{\frac{1}{3}},\tilde{1},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}})$
C_2		$(\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1})$	$(\tilde{1},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\tilde{1})$	$(\tilde{1},\tilde{1},\tilde{2},\tilde{1},\tilde{2},\tilde{2},\tilde{2})$	$(\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\tilde{1})$	$(\widetilde{\frac{1}{3}},\tilde{1},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}})$	$(\widetilde{\frac{1}{2}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{4}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{4}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{3}})$
C_3			$(\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1})$	$(\tilde{2},\tilde{2},\tilde{2},\tilde{3},\tilde{2},\tilde{2},\tilde{3})$	$(\tilde{3},\tilde{5},\tilde{3},\tilde{5},\tilde{3},\tilde{3},\tilde{5})$	$(\tilde{2},\tilde{2},\tilde{3},\tilde{2},\tilde{2},\tilde{3},\tilde{2})$	$(\tilde{2},\tilde{2},\tilde{2},\tilde{3},\tilde{2},\tilde{3},\tilde{3})$
C_4				$(\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1})$	$(\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}})$	$(\widetilde{\frac{1}{3}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{3}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{5}},\widetilde{\frac{1}{2}})$	$(\widetilde{\frac{1}{2}},\tilde{1},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}})$
C_5					$(\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1})$	$(\widetilde{\frac{1}{5}},\widetilde{\frac{1}{5}},\tilde{2},\widetilde{\frac{1}{3}},\tilde{2},\widetilde{\frac{1}{2}},\widetilde{\frac{1}{2}})$	$(\tilde{3},\tilde{2},\tilde{3},\tilde{4},\tilde{5},\tilde{3},\tilde{3})$
C_6						$(\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1})$	$(\tilde{1},\tilde{3},\tilde{3},\tilde{2},\tilde{3},\tilde{2},\tilde{2})$
C_7							$(\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1},\tilde{1})$

Table 5
f_i^+ and f_i^- of inputs.

	C_1	C_2	C_3	C_4
f_i^+	(0.156557, 0.000156629)	(0.165296, 0.000146387)	(0.17529, 0.000162396)	(0.164826, 0.000154162)
f_i^-	(0.25391, 0.000156629)	(0.260051, 0.000146387)	(0.251264, 0.000162396)	(0.260848, 0.000154162)

Table 6
f_i^* and f_i^- of outputs.

	C_5	C_6	C_7
f_i^+	$(0.273498, 4.26909 \times 10^{-5})$	$(0.29375, 6.27114 \times 10^{-5})$	$(0.279775, 4.64215 \times 10^{-5})$
f_i^-	$(0.151927, 4.26909 \times 10^{-5})$	$(0.149209, 6.27114 \times 10^{-5})$	$(0.129739, 4.64215 \times 10^{-5})$

super-efficiency models were also designed to incorporate the idea that DEA efficiency measurements can be sensitive to stochastic variations in inputs and outputs. Stochastic super-efficiency models, such as the one described below, account for the fact that a DMU deemed to be equally efficient in relation to other DMUs may be considered inefficient when random variations in inputs and outputs are introduced (Khodabakhshi, Asgharian, & Gregoriou, 2010).

Let $\tilde{x}_j = \{\tilde{x}_{1j}, \ldots, \tilde{x}_{mj}\}$ and $\tilde{y}_j = \{\tilde{y}_{1j}, \ldots, \tilde{y}_{sj}\}$ be random inputs and outputs related to $\text{DMU}_j (j = 1, \ldots, n)$. Let also $x_j = \{x_{1j}, \ldots, x_{mj}\}$ and $y_j = \{y_{1j}, \ldots y_{sj}\}$ be the expected values of inputs and outputs for DMU_j. The deterministic equivalent of a CCR-based stochastic

Table 7
Values of S_j, R_j and Q_j.

Branch number	S_j	R_j	Q_j
A_1	0.557496	0.203533	0.68982
A_2	0.744393977	0.235194974	0.97959
A_3	0.703039	0.196614	0.89965
A_4	0.19436	0.0683	0.26266
A_5	0.546335	0.162985	0.70932
A_6	0.410917	0.153884	0.5648
A_7	0.336302386	0.072058693	0.40836
A_8	0.666043	0.195965	0.86201
A_9	0.601607	0.222285	0.82389
A_{10}	0.693521	0.179332	0.87285
A_{11}	0.422593	0.131185	0.55378
A_{12}	0.743061486	0.224100597	0.97316
A_{13}	0.734925	0.201072	0.936
A_{14}	0.453907	0.148905	0.60281
A_{15}	0.254321	0.072397	0.32672
A_{16}	0.395389	0.17919	0.57458
A_{17}	0.397686127	0.186185505	0.58387
A_{18}	0.2652	0.154843	0.42004
A_{19}	0.450758	0.17379	0.62455
A_{20}	0.517903	0.118685	0.63659
A_{21}	0.628836	0.256274	0.88511
A_{22}	0.52806143	0.174139567	0.7022

Table 8
Stochastic super-efficiency of the different bank branches.

Branch number	Efficiency value
A_1	0.980447
A_2	0.729166
A_3	0.665765
A_4	1.278485
A_5	0.812143
A_6	1.124623
A_7	0.93884
A_8	0.773272
A_9	0.836179
A_{10}	0.758504
A_{11}	1.243713
A_{12}	0.732622
A_{13}	0.823871
A_{14}	0.779882
A_{15}	1.105976
A_{16}	0.996153
A_{17}	1.115669
A_{18}	1.159669
A_{19}	1.084312
A_{20}	0.86007
A_{21}	0.97071
A_{22}	1.062585

input-oriented super-efficiency model is given by model (19) (see Eq. (10) in Khodabakhshi et al. (2010)):

$$\begin{aligned}
&\min \theta_0^S \\
&s.t. \\
&\sum_{\substack{j=1\\ j\neq 0}}^{n} \lambda_j x_{ij} + s_i^- - \phi^{-1}(\alpha) w_i^I = \theta_0^S x_{i0}, \quad i = 1,\dots,m \\
&y_{r0} - \sum_{\substack{j=1\\ j\neq 0}}^{n} \lambda_j y_{rj} + s_r^+ - \phi^{-1}(\alpha) w_r^0 = 0, \quad r = 1,\dots,s \\
&(w_i^I)^2 = \sum_{j\neq 0}\sum_{k\neq 0} \lambda_j \lambda_k cov(\tilde{x}_{ij},\tilde{x}_{ik}) - 2\theta_0^S \sum_{j\neq 0} \lambda_j cov(\tilde{x}_{ij},\tilde{x}_{i0}) + (\theta_0^S)^2 var(\tilde{x}_{i0}) \\
&(w_r^0)^2 = \sum_{k\neq 0}\sum_{j\neq 0} \lambda_k \lambda_j cov(\tilde{y}_{rk},\tilde{y}_{rj}) - 2\sum_{k\neq 0} \lambda_k cov(\tilde{y}_{rk},\tilde{y}_{r0}) + var(\tilde{y}_{r0}) \\
&s_i^-, \lambda_j, s_r^+, w_i^I, w_r^0 \geqslant 0
\end{aligned} \tag{19}$$

where ϕ is the cumulative distribution function of a standard Normal random variable and ϕ^{-1} is its inverse. x_{ij} and y_{rj} are assumed to be the means of the input and output variables, which, in our case study, correspond to the averages of the values of the inputs and outputs observed during 8 weeks. θ_0^s is the stochastic super-efficiency of DMU_0 and s_i^-, s_r^+ denote the corresponding input and output slack variables, respectively. λ_j is the reference weight for $\text{DMU}_j (j = 1,\dots,n)$, with $\lambda_j > 0$ implying that DMU_j is used to construct the composite unit for DMU_0. Finally, w_i^I and w_r^o are non-negative variables (Khodabakhshi et al., 2010).

We have used LINGO software to perform the above minimization problem applied to the numerical values of our case study. The efficiency of the alternatives derived from the stochastic super-efficiency model defined in Eq. (19) is presented in Table 8.

The rankings of the bank branches derived from our stochastic VIKOR method and the super-efficiency DEA model are described in Table 9, allowing for an intuitive comparison between them.

In broad terms, the ranking results obtained from our VIKOR method and the stochastic version of DEA are not significantly different, particularly when considering the higher and lower levels of the ranking. On the other hand, some variability is expected to arise between both rankings, since our model is highly dependent on the (subjective) criteria weights defined by the experts and the subjectivity inherent to the evaluation of the DM through the choice of the v parameter. The subjective variability introduced in our model can distort what may be considered as the objective evaluation of DEA, which concentrates on the distributional properties of the observations when calculating the efficiency of the DMUs and does not allow for personal judgments. Note, however, that the latter ones are particularly relevant when not all selection criteria are considered to be equally important.

Table 9
Ranking of bank branches: stochastic VIKOR vs. super-efficiency DEA.

Branch number	Stochastic VIKOR ranking	Stochastic super-efficiency ranking
A_1	12	10
A_2	22	21
A_3	19	22
A_4	1	1
A_5	14	16
A_6	6	4
A_7	3	12
A_8	16	18
A_9	15	14
A_{10}	17	19
A_{11}	5	2
A_{12}	21	20
A_{13}	20	15
A_{14}	9	17
A_{15}	2	6
A_{16}	7	9
A_{17}	8	5
A_{18}	4	3
A_{19}	10	7
A_{20}	11	13
A_{21}	18	11
A_{22}	13	8

In order to verify the above intuition formally, we compute the existing correlation between both rankings. As Table 10 illustrates, the Spearman rank correlation coefficient equals 0.815 and is significant, verifying the similarity between our results, which account for the uncertainty and subjectivity inherent to the analysis, and those of the super-efficiency DEA model.

Table 10
Correlation between the stochastic VIKOR and super-efficiency DEA rankings.

Spearman's rho		DEA rank	VIKOR rank
DEA rank	Correlation coefficient	1	.815[a]
	Sig. (2-tailed)		.000
	N	22	22
VIKOR rank	Correlation coefficient	.815[a]	1
	Sig. (2-tailed)	.000	
	N	22	22

[a] Correlation is significant at the 0.01 level (2-tailed).

7. Conclusion

The VIKOR method was originally developed as a MCDM technique to determine a preference ranking from a set of alternatives in the presence of conflicting criteria. In this paper, we have introduced an extended VIKOR method applicable to problems with stochastic data. The proposed approach has been applied to a case study for evaluating the performance efficiency of 22 bank branches based on 7 different criteria.

The stochastic VIKOR method provides a more complete picture of the decision making process, enabling the DMs to select the alternative that is more in accordance with their subjective interests. The main advantage of using the stochastic VIKOR method is that it allows us to take into account the variability of the real world when determining the value of each criterion. The proposed method has also an advantage over the traditional theory when measuring the ambiguity of concepts that are associated with the subjective judgments of human beings.

We have illustrated how the information provided by the extended VIKOR method can be used to design different development plans for an alternative to achieve a higher position in the final ranking. Moreover, the extended method proposed is sufficiently flexible so as to provide a systematic approach that can be easily applied to other MCDM techniques such as PROMETHEE and TOPSIS with stochastic data.

One of the limitations of our extended method is that it has been defined assuming that the decision criteria are independent. An immediate extension of this research could consist of developing a hierarchical stochastic version of the original VIKOR model.

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